STRESS CONCENTRATION FACTORS FOR CIRCULAR NOTCHES

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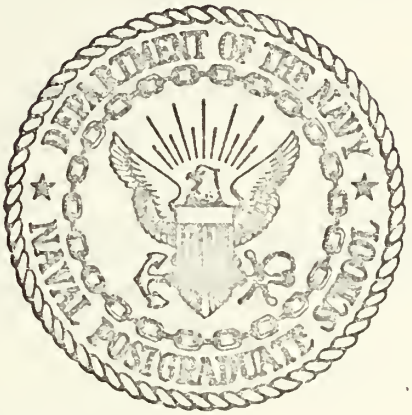

THESIS

STRESS CONCENTRATION FACTORS
FOR CIRCULAR NOTCHES
by
Lawrence Jerome Smith

June 1974

Thesis Advisor: J.E. Brock

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Existing finite element programs PLISOP and PLIMEG are modified and adapted to the successful determination of stress concentration factors for thin elastic bars containing V-shaped round bottom notches and subjected to tensile or bending loads. Accuracy and reliabilty are demonstrated. Comparisons are made with previous analytical and experimental determinations. The effects of finite bar thickness are discussed.

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Stress Concentration Factors for Circular Notches

by

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Existing finite element programs PLISOP and PLIMEG are modified and adapted to the successful determination of stress concentration factors for thin elastic bars containing V-shaped round bottom notches and subjected to tensile or bending loads. Accuracy and reliability are demonstrated. Comparisons are made with previous analytical and experimental determinations. The effects of finite bar thickness are discussed.

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I. INTRODUCTION

Stress intensifiers, in the form of holes, grooves, keyways, threads, etc., result in local modification of simple stress patterns and give rise to stress concentrations. The designer or analyst is interested in determining the locally maximum, governing or limiting value of stress, designated by the symbol σ_{max}, which is calculated as the product of a nominal stress, σ_{nom}, and a stress concentration factor, designated by the abbreviation SCF, thus:

$$\sigma_{max} = SCF \times \sigma_{nom}$$

An elementary formula is used to compute σ_{nom}, the terms of which must be precisely defined; this accounts for the gross geometry of the member and the loading which is applied. The SCF is intended to provide a corrective multiplying factor; its value depends on the details of local geometry and, usually, on the particular type of loading that is applied.

The designer employs available tabulations of SCF's, together with the appropriate and easily calculated σ_{nom}, employing the formula given above, to estimate the maximum stress which he compares with an allowable value; the latter being established by experience, by code, or by contract.

The task of the person who supplies values of SCF is to determine values of σ_{max} and of σ_{nom} and their quotients, the

SCF's. To make the values of SCF most widely useful, the stress analyst selects an appropriate set of parameters to describe the geometry of a class of stress intensification problems. A number of individual problems are undertaken, taking care to vary the parameters so as to cover the useful range of each and thus to permit making reasonable interpolations for intermediate values.

The SCF's obtained in this manner are usually presented in the form of parametric curves drawn through the actual data points established. The designer then must interpolate between the curves to match the actual parameters of his problem.

The particular classes of problems selected for this study, because of their prominence and importance, are characterized by a thin, flat bar under tension and/or bending. We consider two forms of stress intensifiers:

1. Two notches on opposite edges of the bar, located symmetrically about the long axis of the bar and shaped symmetrically with respect to a plane normal to the long axis of the bar.
2. A notch on only one edge of the bar.

In general, the notches are circular at the bottom. Details of the notches, with sketches, are presented in Section III. Much of what follows, however, is applicable to other geometries.

We confine attention to the two-dimensional case, i.e., the case of plane stress. Realizing that a condition of plane stress is never actually found in a real world application, the effect of three dimensionality on SCF's was also investigated, see Appendix C.

II. BASIC METHODS OF OBTAINING SCF'S

A. CLASSICAL ANALYSIS

Neuber [11]has made the most extensive analytical investigation of SCF's. His book contains treatments of a number of geometries and loadings and his methods include a few cases of exact analytical solutions together with a number of ingenious approximations and interpolations. The best known of Neuber's analyses is that of the deep hyperbolic notch.

1. Neuber's Notch Analysis

Using curvilinear coordinates and complex stress functions, Neuber presented a solution for axial tensile loading (without bending) of a geometry consisting of symmetrically located, hyperbolic shaped notches in an infinite sheet of unit thickness (Fig. 1).

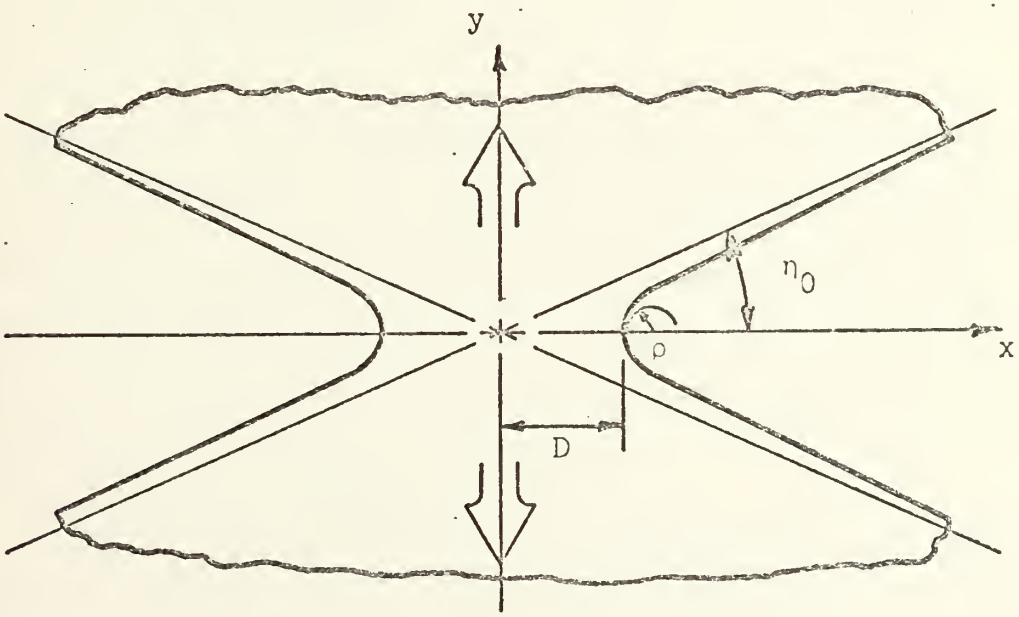


Fig. 1. Symmetric hyperbolic notches in an infinite sheet.

He arrived at the following solution for the stress concentration factor for this notch:

$$SCF = \frac{4 \cot \eta_0}{\pi - 2\eta_0 + \sin(2\eta_0)}$$

where η_0 is a parameter defining the particular notch geometry, namely, the angle between asymptote and notch centerline. The nominal stress is given by the axial force divided by the minimum width of the sheet at the notch. The hyperbolic boundary is described by the equation:

$$\frac{x^2}{a^2 \cos^2 \eta_0} - \frac{y^2}{a^2 \sin^2 \eta_0} = 1$$

The half width at the bottom of the notch is D. The minimum radius of curvature is

$$\rho = D \tan^2(\eta_0) \quad .$$

The width of the sheet approaches infinity on each side, unlike cases of practical interest in which the notch is an indentation in a bar of finite width. Accordingly, for Neuber's deep hyperbolic notch, it is not possible to specify a gross width or the ratio of gross width to net width as is usually done for practical cases.

Neuber further presents an approximation for shallow notches in a bar of finite width with notch depth specified.

The analysis involves employing a solution for an elliptic hole in a bar. Somewhat arbitrarily the bar is cut through the center of the hole parallel to the edges of the bar and reassembled to form the notched bar (Fig. 2).

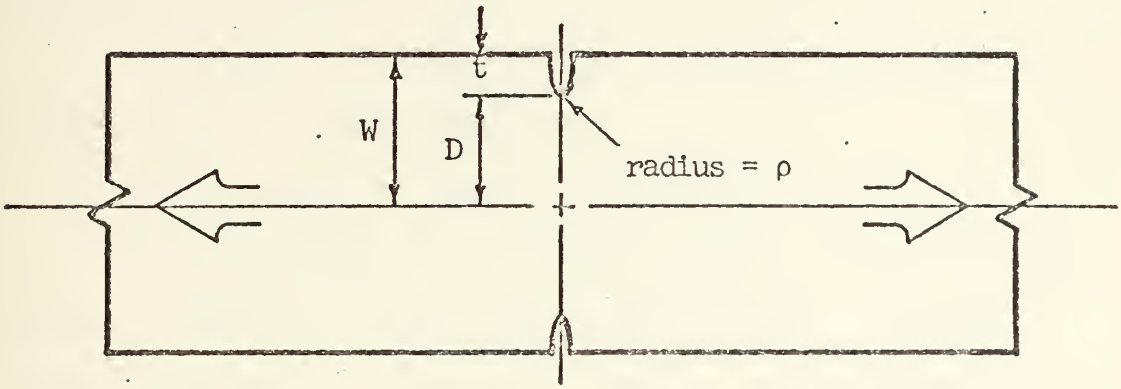


Fig. 2. Neuber's shallow notch approximation

The boundaries of the notched bar are not unloaded as they should be and there are unaccounted for dislocations along the axial center line.

Taking the half width of the bar as W and using D, ρ, and σ_{nom} as defined for the hyperbolic notch analysis, Neuber arrived at the following approximation for stress concentration factor for the shallow notch:

$$SCF = 1 + 2[(W - D)/\rho]^{\frac{1}{2}}$$

Noting that nominal stress was defined the same way in both cases, i.e., on the basis of net area, Neuber devised

a nomographic scheme to obtain SCF for a wide range of the parameters:

ρ = minimum radius of curvature

D = half width of the bar at the notch (net half width)

W = half width of the undisturbed bar (gross half width)

The nominal stress is σ_{nom} =(axial force per unit thickness)/2D.

A summary of Neuber's solution for the hyperbolic notch is shown in Appendix B.

2. Limitations of Neuber's Analysis

The hyperbolic notch analyzed has a definite shape, namely a hyperbola having the asymptotes $y = \pm x \tan \eta_0$. The values presented by Neuber are based on two extremes and an interpolation between them. It must be recognized that other geometries only approximate these notch shapes and therefore the data must be judiciously applied.

The reasoning used to arrive at this solution is ingenious but experimental evidence described later indicates a systematic difference between Neuber's analysis and the behavior of actual test sections.

3. Availability of Neuber's Solution

A designer may use Neuber's equations and perform the indicated interpolation to determine a particular SCF based on the parameters of interest. However, much of that work has already been done.

The most extensive, readily available tabulation of SCF for this case is presented by Peterson [12]. He presents a family of curves using dimensionless ratios of the

parameters as arguments. Other authors, such as Faires [6] present the same data over a narrower range of parameters.

B. EXPERIMENTAL INVESTIGATION

Values of maximum stress may be determined by experimental stress analysis including strain gage measurements and photoelastic evaluations.

1. Strain Gage Measurements

Kikukawa [9] used resistance strain gages to study the stress in a bar with two symmetrically located U-shaped notches subjected to an axial tensile load.

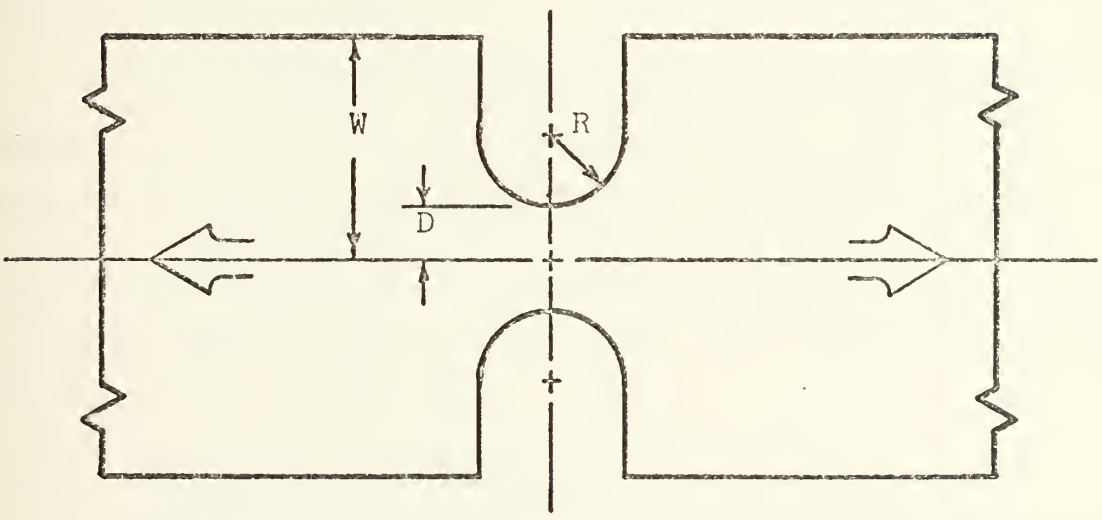


Fig. 3. Symmetric U-shaped notches

The geometry of the test section (Fig. 3) is specified by:

R = radius at the bottom of the notch

D = net half width

W = gross half width

Only the gross width of the bar was varied during the tests.

2. Limitations of Strain Gage Measurements

In using strain gages, difficulty arises from the highly localized character of the stress distribution at the place in question. A very small gage length is necessary to obtain satisfactory results. Accordingly, the physical size of available strain gages limits their usefulness to the investigation of relatively large notches. Furthermore, models must be very carefully machined to obtain dimensions within satisfactory tolerances. Good adhesion of the gage to the model and correct placement and orientation on the specimen are difficult to obtain.

Data scatter dictates that some sort of smoothing process must be used before the results are useful to a designer. Large expenditures of time and other valuable resources are required to obtain even a few benchmarks which can reasonably be used to infer SCF's over some useful range of the parameters employed.

3. Photoelastic Evaluations

Flynn and Roll [7] used photoelastic techniques to study the stress in a bar subjected to an axial tensile load for both U-shaped and hyperbolic-shaped symmetrically located notches. The geometry of the U-notched test section (Fig. 3) is specified by:

R = radius at the bottom of the notch

D = net half width

W = gross half width

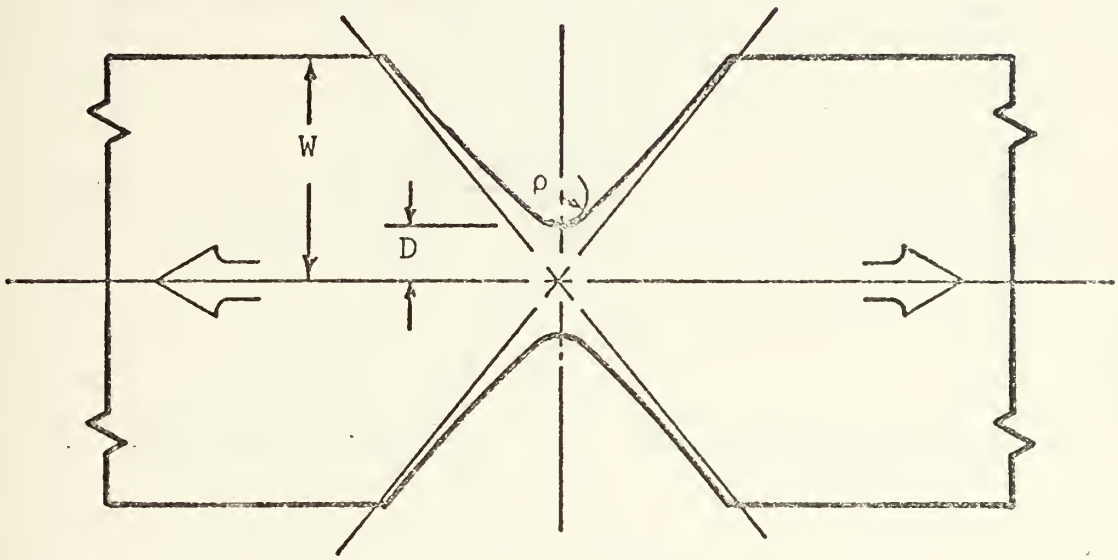


Fig. 4. Symmetric hyperbolic notches

The geometry of the hyperbolic-notched test section (Fig. 4) is specified by:

ρ = minimum radius of curvature

D = net half width

W = gross half width

The gross width of the bar was the only parameter which was varied in these tests, also.

Appl and Koerner [2] used photoelastic techniques to study the stress in a notched bar subjected to an axial load. The notches were symmetrically located, V-shaped with various angles and had round bottoms.

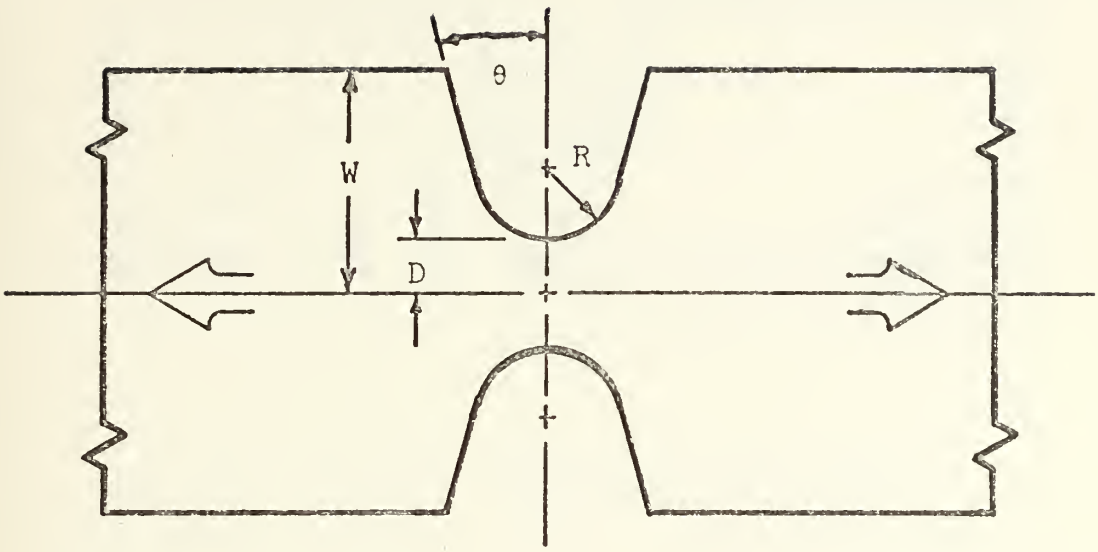


Fig. 5. Symmetric V-shaped notches

The geometry of the test section (Fig. 5) is specified by:

R = radius at the bottom of the notch

D = net half width

W = gross half width

θ = half included angle

They present SCF's for several notch angles using the ratio D/W as a parameter.

Cole and Brown [5] used photoelastic techniques to study the stress in a bar with a U-shaped notch on only one side of the bar. An axial load was applied such that its line of application was through the center of the net cross section.

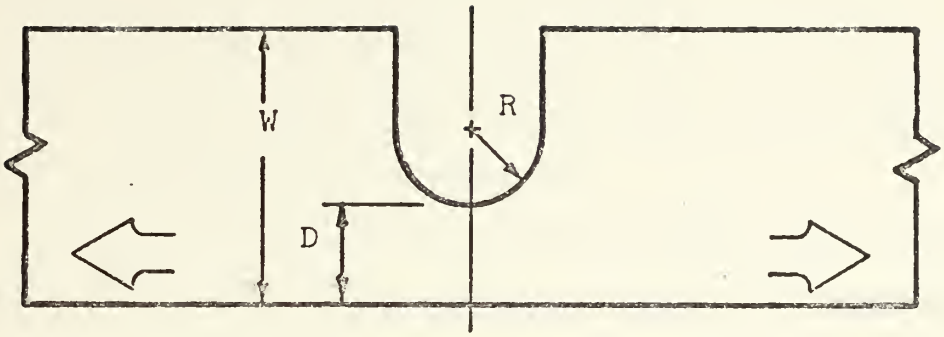


Fig. 6. U-shaped notches, one side only

The geometry of the test section (Fig. 6) is specified by:

R = radius at the bottom of the notch

D = net width

W = gross width

They present SCF's for several combinations of the parameters.

4. Limitations of Photoelastic Evauluation

Models, no matter how thin, do have a third dimension. These experimenters have chosen thin models purposely so that their results could be reported as plane stress values. Appendix C discusses the effect of the third dimension, showing that the stress at the base of the notch actually varies through the thickness. Photoelastic evaluation reveals the stress

averaged through the model thickness. The difference between the averaged and true maximum is small, but the three dimensional effect should be recognized.

Errors due to several edge effects including those caused by residual machining stresses and time dependent material changes must be recognized.

The same limitations regarding model preparation, data scatter and resource expenditure that were mentioned as limitations to strain gage measurements apply also to photoelastic evaluations.

C. NUMERICAL TREATMENT OF ELASTIC FIELD EQUATIONS

Several numerical techniques, notably the Finite Difference method, are available and have been used to solve partial differential equations. However, useful results to the elastic field equations are difficult to obtain, especially if the geometry of the problem is not very simple.

Recently a new numerical method called the Finite Element Method (FEM) has been extensively developed and used. In many ways it is similar to earlier numerical methods in that the partial differential equations are replaced by a finite system of algebraic equations. There is, however, a big difference in the detail of this replacement and this difference in detail permits the user to deal with complex geometric shapes.

The method has been well proved by the solution of problems which have classically known solutions. Thus it

seems to provide a new and powerful way to determine stress concentration factors.

Considering all of the above, the Finite Element Method (FEM) was chosen for use in this study.

III. DETAILED PROBLEM SPECIFICATION

A. GENERAL

Before proceeding with FEM program selection, it will be convenient to have the problem specified in full detail. This is done so that further reference to computer program selection and modification can be with the requirements of this problem specifically established.

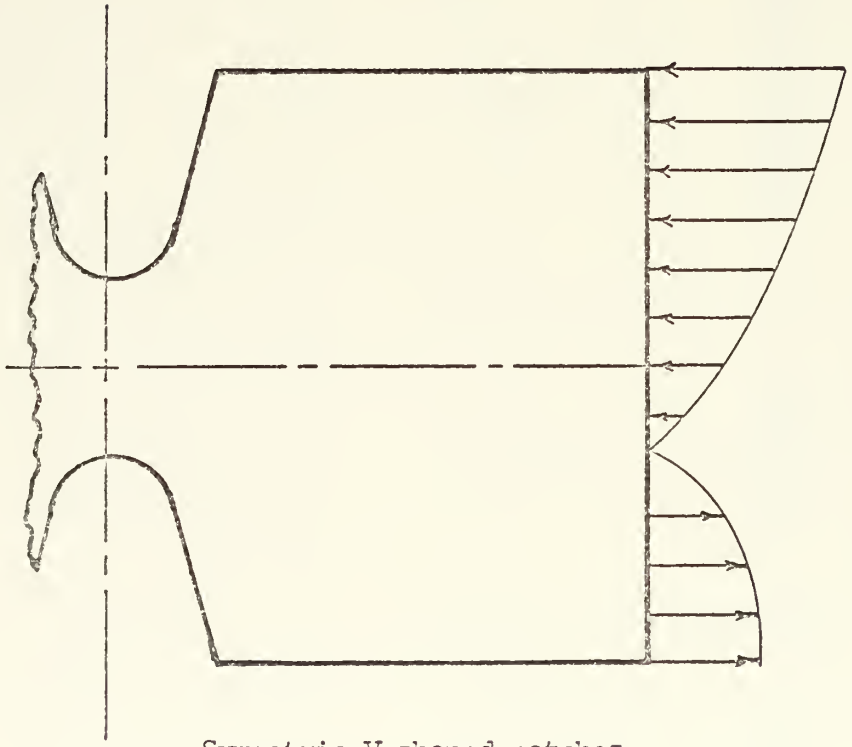
In Section I we limited the study to a family of thin, flat bars with general end loading. The stress intensifiers were limited to two forms: a single notch and two symmetrically located notches. Now specifically we deal with that general loading and a detailed description of notch shape. A single notch shape is considered: the bottom of the notch is circular, characterized by its radius, and the sides are V-shaped. The included angle of the V may be between 0° (U-shaped notch) and 180° (unnotched bar).

This problem family was chosen because of its practical importance and because it includes much of the available experimental data.

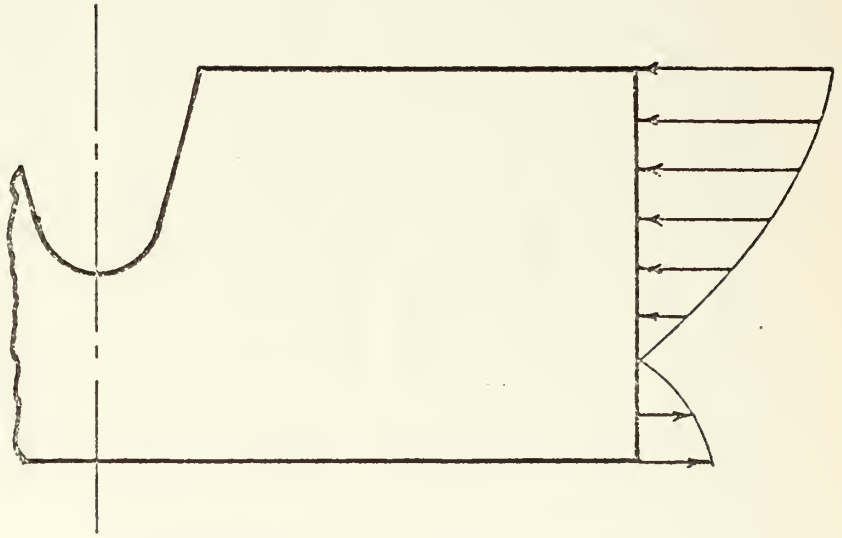
B. FOUR PROBLEMS

Division of the problem family into two general classes based on the number of notches has already been indicated. The two classes are shown in Fig. 7.

We speak of general loading at the ends of the bar containing the stress intensifier. However, by virtue of the



Symmetric V-shaped notches
with general load



V-shaped notch on one side only
with general load

Fig. 7. Two Problem Classes

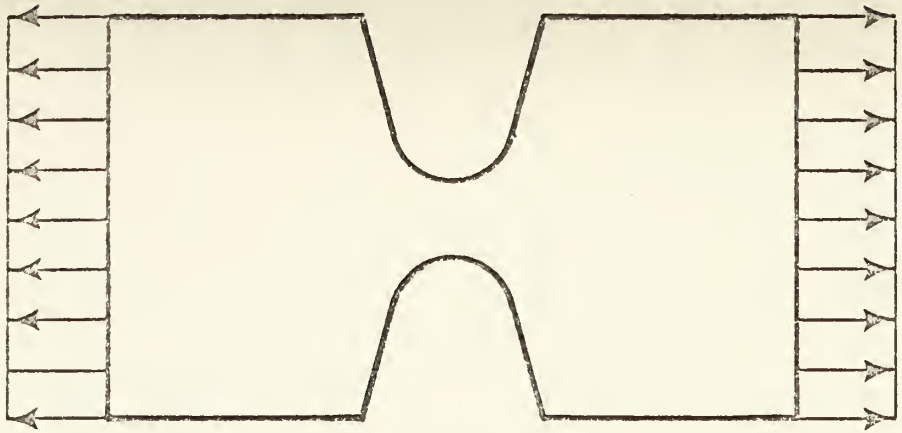
principle of Saint Venant, if the bar is several times longer than its width, as we assume and as is the case in practical applications, then only the static properties of the load, i.e., the net axial force and the moment (about, say, the central axis of the bar) are of importance. Thus what we have called a general loading can be treated by the superposition, in proper proportion, of a case with an axial force and no moment and a case with moment and no net axial load. Based on this, the four specific problems shown in Fig. 8 and described in Table I were selected for study.

TABLE I

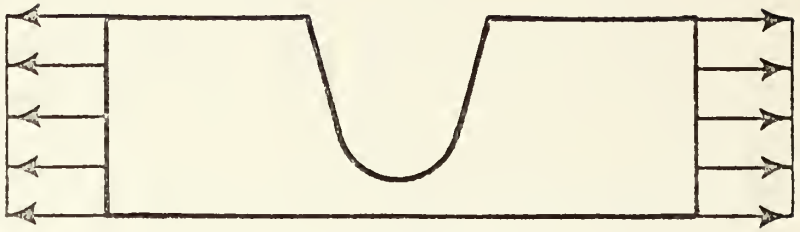
Problems Studied

Problem Number	Notches	Loading
1	Two	Axial Force
2	Two	Moment
3	One	Axial Force
4	One	Moment

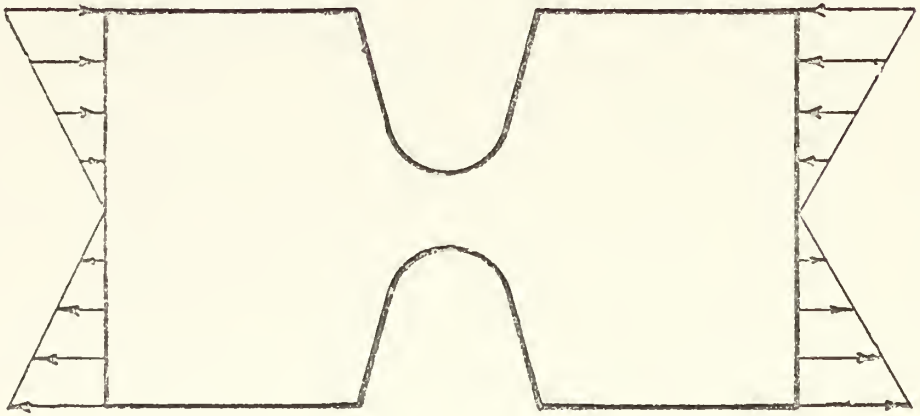
The general loading of the class of problems with two notches can be obtained by an appropriate superposition of problems 1 and 2. The maximum stress (σ_{max}) always occurs at the base of one of the two notches and since they are symmetric, it is apparent that the maximum stress can always be calculated.



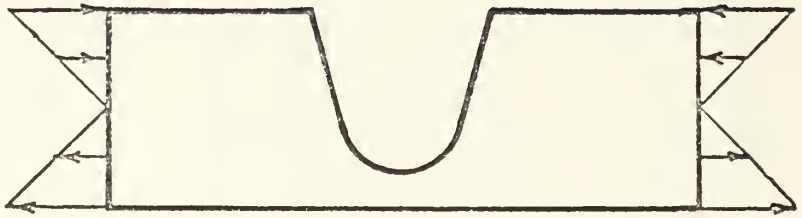
Problem 1



Problem 3



Problem 2



Problem 4

Fig. 8. The Four Problems

The general loading of the class of problems with one notch can similarly always be obtained by an appropriate superposition of problems 3 and 4. However the maximum stress does not always occur at the base of the notch. Consider first the bending of problem 4. Maximum tensile stress occurs on the straight boundary opposite the notch and maximum compressive stress at the bottom of the notch. Superimposing problem 3 can bring the stress at the bottom of the notch to any value, say zero, producing maximum stress on the boundary opposite the notch.

Accordingly, for this class, data should be generated and made available both for the point at the bottom of the notch and also for the opposite point on the straight boundary.

C. ONE STRUCTURE

By taking advantage of symmetry, the four problems may be reduced to four cases each of which is completely described by the single geometry of Fig. 9 and one of four different sets of boundary conditions. Face B is a circular arc of radius R. Face C is a straight line inclined at angle θ. The other four faces, A, D, E and F, are straight lines parallel to the x or y axis. The length of face A is designated D, the length of face E is designated W, and the length of face F is designated L. We take $L = 5W$ so as to assure, by St. Venant's principle, that fine details of end loading do not influence the stress distribution in the notched portion of the bar.

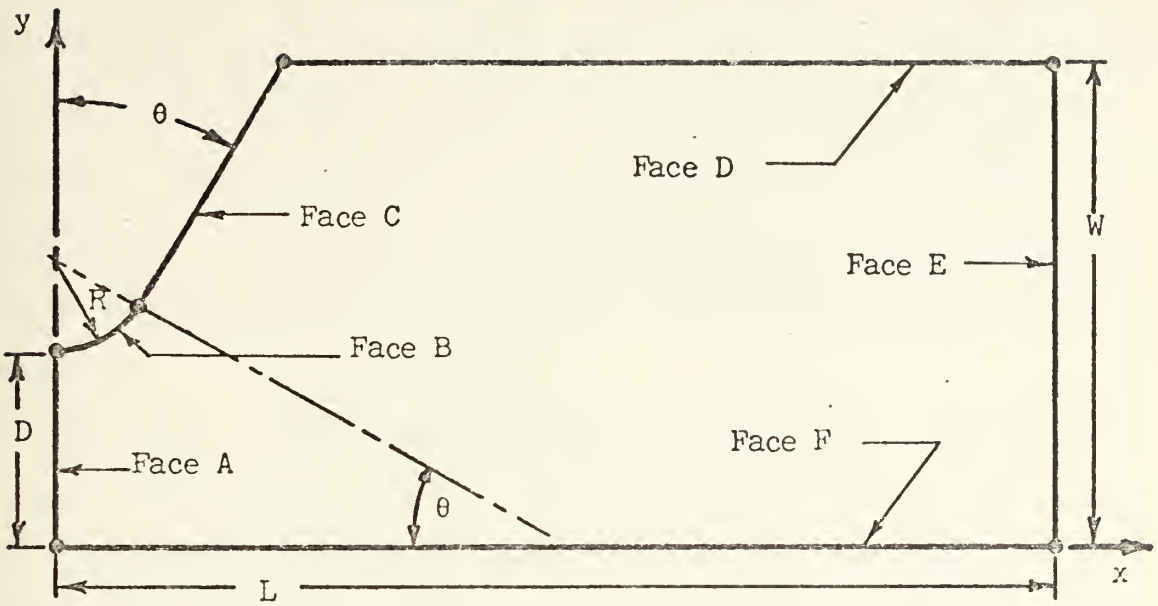


Fig. 9. Structure faces

The geometry of the structure can now be adequately described by the dimensionless ratios $\frac{R}{W}$, $\frac{D}{W}$ and θ .

D. FOUR SETS OF BOUNDARY CONDITIONS

The four cases to be described here and summarized in Tables II and III are equivalent to the four problems described earlier. Cases c and d are problems 3 and 4 directly. Appropriate superposing of two cases a and of two cases b along face F yield problems 1 and 2 respectively.

At this point we delineate the boundary conditions corresponding to each of the four cases. It is evident that we have to specify values of the displacements u and v along the faces A, B, C, D, E and F and/or the values of the forces

F_x and F_y applied along these faces by external agencies. Along faces B, C and D there are no externally applied forces and the displacements u and v are not specified (that is, they are unknowns which will be solved for by the FEM program). All internal points of the structure, likewise, have no specified values of u or v and are not externally loaded.

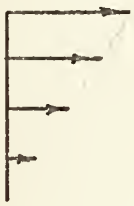
Because of the symmetry in all four cases, we specify that u = 0 all along face A. Along face F, for cases c and d, there are no boundary forces and values of u and v are unspecified. For case a, face F is along a line of symmetry and we take v = 0 all along face F. For case b, the condition of antisymmetry is satisfied by taking u = 0 at every point along face F. In neither of these cases are there any prescribed boundary forces along face F. In all four cases, the point which marks the intersection of faces A and F is restrained so that u = v = 0.

Loadings are applied on face E. The displacements along face E are unspecified. All forces F_y are zero. Values of F_x along this face are such that they produce the required loadings as shown in Table II.

A summary of boundary conditions for faces A, B, C, D and F is given in Table III. The conditions are given as triples (u,v,F_x). All values of F_y are zero. An asterisk (*) indicates that no value is specified.

TABLE II

EXTERNAL LOADING ALONG FACE E

Loading	Case Number	Static Effect	
		F_x	M
	a	P	$\frac{PW}{2}$
	b	$-\frac{3P}{2}$	- PW
	c	P	0
	d	0	- PW

The displacements along face E are unspecified. All forces F_y are zero.

TABLE III

BOUNDARY CONDITION SPECIFICATION

Face	Case 1	Case 2	Case 3	Case 4
A	(0,*,*)	(0,*,*)	(0,*,*)	(0,*,*)
B,C,D	(*,*,0)	(*,*,0)	(*,*,0)	(*,*,0)
F	(*,0,0)	(0,*,*)	(*,*,0)	(*,*,0)

IV. SOFTWARE SELECTION

A. BASIC PROGRAM SELECTION

The purpose of the study was not only to obtain some useful data (i.e., SCF's) to supplement and refine that already available through other sources, but also to perfect computer software for generating such data and to establish its reliability. Reliable, efficient programs capable of solving the two-dimensional plane stress problem and three-dimensional stress problem have been written and are available. Some of them are proprietary but many appear in the literature and portions of others have been published. Unfortunately, mainly due to the state of the art in the field of computer science, most programs are machine dependent and therefore could not be considered for use in this investigation.

The two programs selected were developed at the Naval Postgraduate School. PLISOP [4] is a two-dimensional finite element program capable of solving the plane stress problem. It utilizes isoparametric elements which means that curved boundaries can be represented. All data handling in this program is confined to the main core storage of the computer. The program has internal documentation which includes instructions for its use. TRISOP [10] is a three-dimensional finite element program utilizing isoparametric elements. Because of the large amount of data implicit in the three-dimensional analysis, much of the program coding deals exclusively with

handling the data. Most of the storage is outside the computer main core on direct-access devices which provides for the storage of large volumes of data. The price paid for this expanded storage is the large computational time needed. Internal documentation includes instructions for its use.

Prior to the start of this investigation both of these programs had been debugged on the IBM-360 computer available for use in this study and solutions to a number of problems had been compared to the classically obtained solutions to demonstrate that the programs are capable of producing very good approximations when used correctly. The remainder of this section deals specifically with the two-dimensional program. Cf. Appendix C for remarks concerning the use of TRISOP.

B. MESH GENERATION

The first step in employing a FEM program for the solution of a specific problem is to provide appropriate input data, describing the problem, in the form called for by the specifications of the particular FEM program employed. This requires the preparation of a very large volume of data. Material properties and program options must be selected, of course, but the great preponderance of the input has to do with geometry. The user must discretize the structure into elements and describe these elements and their interconnection to the program. Coordinates for each node used (specific points on the element boundaries) must be provided along with load boundary conditions. The displacement boundary conditions constitute the third large block of information

required. The elements and nodes are numbered for identification using numbering conventions dictated by the coding of the FEM program. Specified data card format must be adhered to rigidly. The discretization ultimately selected for most of this study would require over 600 data input cards.

Manual preparation of such a volume of data is unthinkable. However this problem is not new and others have dealt with it before and have provided programs to accomplish the task. Such a program is called a mesh generator. It is written to accept a gross description of the structure under consideration (Fig. 12) and its basic function is to superimpose a grid, establishing the elements (Fig. 11) required. Coordinates of all nodes are calculated. Elements and nodes are numbered according to the criteria of the FEM program which the mesh generator serves. The program output is in the form of a deck of punched data cards. The deck, compatible with the FEM program, is in two parts: element descriptions (element connectivity cards) and nodal coordinates (element node cards).

PLIMEG is a computer program originally written by J.R. Adamek as part of his NPS thesis [1] for the purpose of simplifying the preparation of input to PLISOP. Modifications to Adamek's program were made as a part of the present study in order to make it more useful for the purpose of inputting data for the notch stress intensification problem and making optimal use of the computer facilities in solving the problem.

The basic input to PLIMEG is a description of the geometry of the problem, divided, by the author, into several large subdivisions, referred to as superelements, or, briefly, as SEL's. The coordinates of the SEL corner points are specified, the connectivity is indicated, and, in an appropriate fashion, the geometry between corner points is specified. Other inputs to PLIMEG are m and n, the number of subdivisions required along two adjacent SEL boundaries. The numbers m and n must be consistent on any boundary common to two SEL's.

Internally, the program PLIMEG now establishes additional nodal points within each SEL, dividing each SEL into m x n smaller elements — the finite elements themselves. The output data deck is compatible with PLISOP. A program option may be used to obtain a graphical output depicting the completed grid.

The subdividing done in PLIMEG is actually accomplished in a non-dimensional coordinate system (ξ,η) and the grid established there is mapped into the SEL by a coordinate transformation.

C. MODIFICATIONS TO PLIMEG

1. Stress Gradient Considered

The structure considered in the study (Fig. 9) is of interest because of the stress concentration. That concentration implies a large gradient which, in turn, demands small elements in the finite element discretization. The use of a uniformly fine mesh is undesirable from the standpoint of the number of equations that must ultimately be solved. Not only

is the solution more time consuming but the maximum number of elements is directly dependent on machine storage capacity. The IBM-360, using PLISOP, imposes an upper limit of slightly over 180 quadratic elements, and even that size of problem can only be considered by special treatment.

Noting that the high stress gradient is very localized, it is possible however, to use small elements in that immediate area and larger elements where the gradient is smaller. By extension, bands of different size elements may be used. This technique has been successfully employed by many users. Pfeifer [13], for example, uses the three-dimensional stress analysis program (TRISOP) in this way to solve the problem of a concentrated force on the boundary of a semi-infinite body. His results compare well with Boussinesq's classical solution.

Accepting the usefulness of graded element size, a logical next step is to devise a way to vary element size progressively as the stress gradient changes. This was accomplished by modifying PLIMEG. Entering the program at the point where the nondimensional grid has been established, the element corner nodes, represented by (ξ_i, η_i) can be systematically moved by use of the relationships:

$$\xi_i = (\xi_{max} - \xi_{min}) \frac{\exp[a_\xi(\xi_i - \xi_{min})] - 1}{\exp[a_\xi(\xi_{max} - \xi_{min})] - 1} - 1$$

$$\eta_i = (\eta_{max} - \eta_{min}) \frac{\exp[a_\eta(\eta_i - \eta_{min})] - 1}{\exp[a_\eta(\eta_{max} - \eta_{min})] - 1} - 1$$

A wide range of element size variation is provided for by allowing the user to specify the exponential coefficients a_ξ and a_η as PLIMEG input values. The intermediate nodes are inserted on the new grid lines thus established, equally spaced between corner nodes (one if quadratic elements are used and two if cubic elements are used). From this point the coordinate transformation and other details of PLIMEG are as originally programmed. Mesh generation as originally programmed and as modified is illustrated in Fig. 10 for a representative general SEL. Note that PLIMEG, as modified, provides the capability of dividing a SEL into any specified number of increments with any specified exponential coefficient (positive, negative, or zero) in both the ξ and η directions, each completely independent from the other.

2. Curved Boundary Node Relocation

Isoparametric elements are used both for the SEL description and the elements in the completed mesh. Three element types are used. Each is capable of representing a different degree of variation along an element boundary. The element types, with their common names and nodal description are shown in Table IV. The maximum degree of polynomial which can be exactly represented by each element is given. If straight line, parabolic, and/or cubic boundaries determine the geometric shape of the structure, they may be exactly represented by proper choice of SEL type. If more complex curves are needed to describe the structure, or if there are discontinuities, additional SEL's can be used to meet the requirements of the particular use.

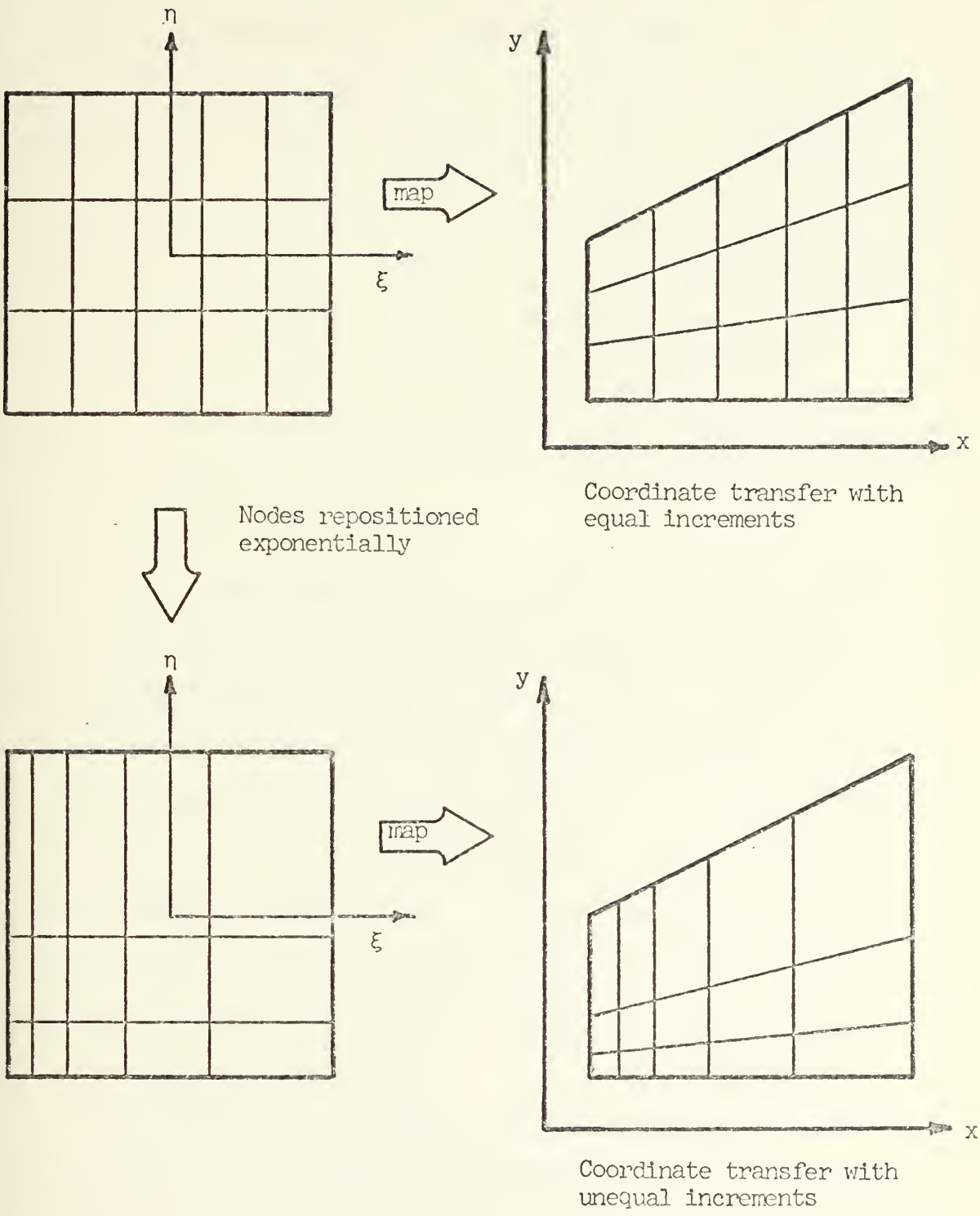


Fig. 10. Mesh generation with and without equal element size

TABLE IV

ISOPARAMETRIC ELEMENT TYPES USED

Common Name	Number of Nodes	Exactly Represents
Linear	Four	Straight Line
Quadratic	Eight	Parabola
Cubic	Twelve	Cubic Polynomial

It is important to note that SEL boundaries which cannot be exactly described by the element type chosen (i.e., are of higher order than the element used to describe them) become slightly distorted in the mapping procedure. Except for those boundaries which coincide with the boundaries of the structure, this is of no consequence. The structure boundary nodes, however, must be corrected to locate them properly in the real coordinate system. A provision to accomplish this for the curved boundary of Face B (Fig. 9) has been included in PLIMEG.

A representative mesh, as used in this study, is shown in Fig. 11. Quadratic elements are used througout this study.

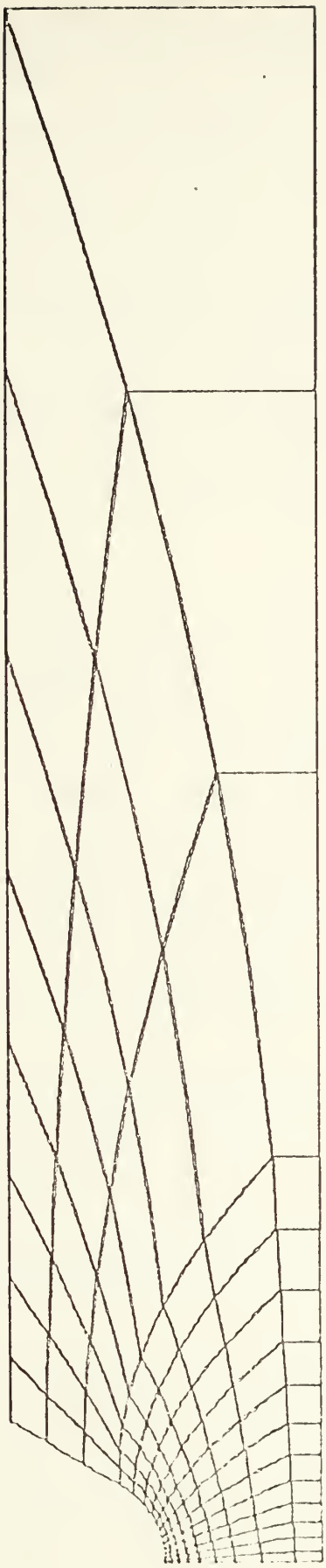


Fig. 11. A sample mesh as used

3. Boundary Condition Calculations

Boundary conditions for the structure are in two categories — force and displacement. As established in Section III D , there are two sets of force conditions and four sets of deflection conditions. These are easily coded into PLIMEG where the appropriate input data for the FEM program is prepared. A particular set of boundary conditions is imposed by a code number which exactly corresponds to one of the four problems outlined in Section III B.

D. SUBROUTINE PROBLM

The test structure, divided into the SEL arrangement utilized, is illustrated in Fig. 12. SEL numbers and SEL node numbers are indicated. The SEL arrangement was selected after considerable experimentation with other arrangements, based on the following considerations:

1. Small elements are required at the base of the notch because of the high stress gradient there.
2. Large elements are adequate away from the notch, specifically at the end of the structure where loads are applied.
3. Storage capacity of the computer limits maximum number of elements to not more than 180.
4. Extreme element corner angles (i.e. near 0° or 180°) must be avoided.
5. Each SEL must be a four-sided figure.
6. Relying on the principle of Saint-Venant, a single element boundary may be used to represent the loaded end.

The coordinates of most of the nodes are obvious after certain key nodes are located. The origin is at node 27. Nodes 1, 4, 7, 20, and 33 are established by the parameters

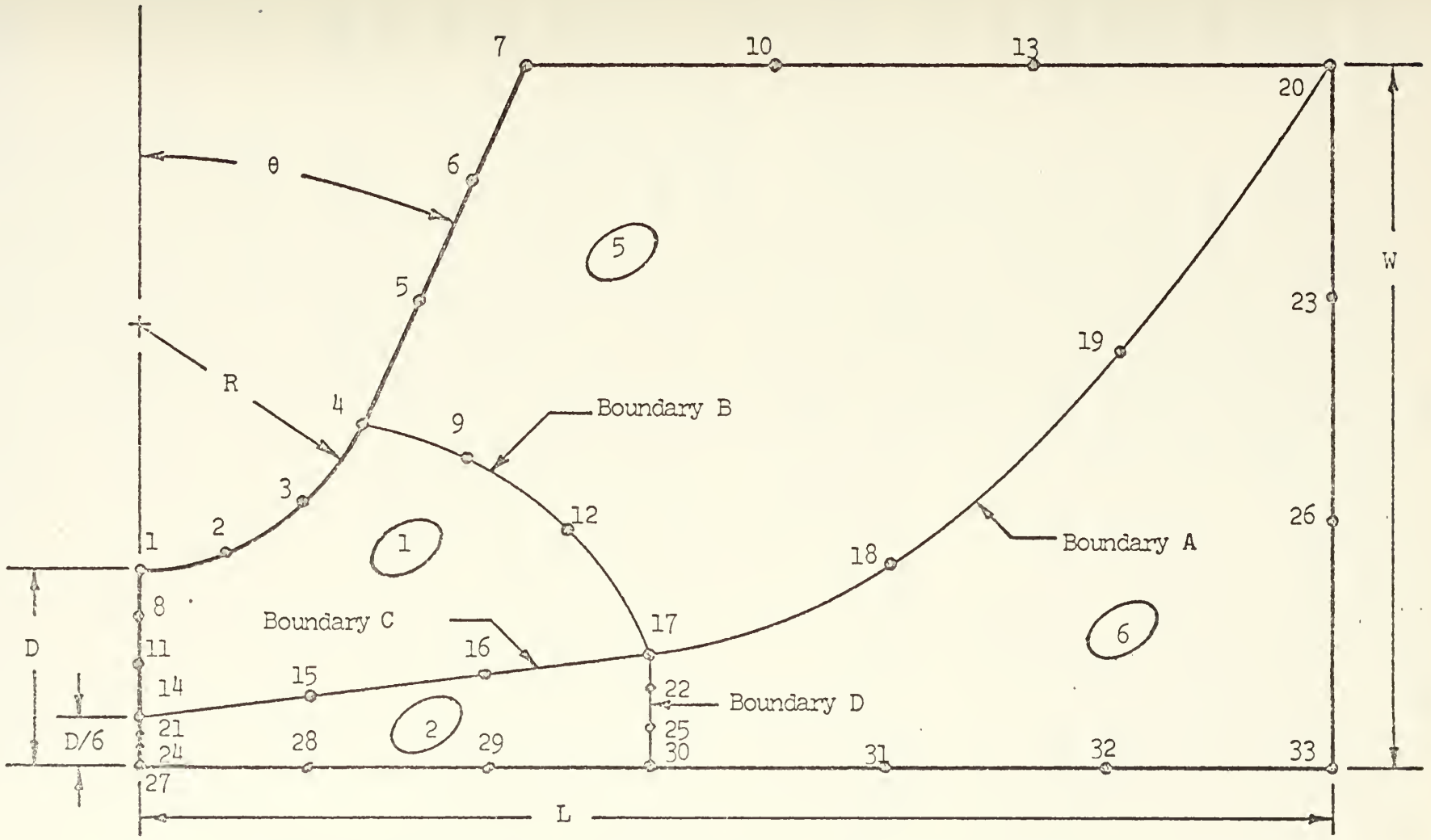


Fig. 12. Structure SEL's and boundary nodes. SEL identification numbers appear in ovals. (Not to scale)

directly (R/W, D/W, θ), when L = 5W is provided. The x coordinate of node 30 was established at W + R and the y coordinate of node 14 at D/6. Boundary A is an arc of the parabola passing through nodes 14 and 20 with zero slope at node 14. Boundary B is an arc of another parabola through nodes 4 and 17 perpendicular to boundary A at node 17. Boundaries C and D are straight lines. Intermediate nodes, where indicated, are located at the 1/3, 2/3 points along the curve establishing the boundary. Table V is a summary of the SEL node coordinates.

The program PROBLM, written to calculate SEL node coordinates uses the above criteria and requires a single card input: parameters R/W, D/W and θ.

E. SUBROUTINE ANS

The answer required for each problem is the stress concentration factor which is calculated in subroutine ANS. This program produces a single card reporting problem parameters and the stress concentration factor associated with them. The stress concentration factors reported are based on the following definitions of σ_{nom}.

Case 1, uniform loading:

$$\sigma_{nom} = P/A$$

where:

P = total load applied *

A = 2D = net width of the bar

*Actually here and elsewhere, loads are per unit thickness.

TABLE V

SEL Node Coordinates

Node	SEL's	x coordinate	y coordinate
1	1	0	D
2	1	$R \sin(30° - \theta/3)$	Note (1)
3	1	$R \sin(60° - 2\theta/3)$	Note (1)
4	1,5	$R \cos \theta$	$(D+R) - R \sin \theta$
7	5	$x_4 + (W-y_4) \cot \theta$	W
9	1,5	$x_4 + (x_{17}-x_4)/3$	Note (3)
12	1,5	$x_4 + 2(x_{17}-x_4)/3$	Note (3)
14	1,2	0	D/6
17	1,2,5,6	R + W	Note (2)
18	5,6	$x_{17} + (x_{20}-x_{17})/3$	Note (2)
19	5,6	$x_{17} + 2(x_{20}-x_{17})/3$	Note (2)
20	5,6	5W	W
27	2	0	0
30	2,6	R + W	0
33	6	5W	0

Notes:

(1) Determined by circular arc of radius R through 1,4.

(2) Determined by parabola A through 14, 20 with zero slope at 14.

(3) Determined by parabola B through 4, 17 perpendicular to parabola A at 17.

(4) Points not listed are on straight line boundaries equally spaced between corner nodes.

Case 2, bending moment:

$$\sigma_{nom} = Mc/I$$

where:

M = total moment applied

c = D = distance from neutral axis

$I = 2D^3/3$ = moment of inertia

Case 3, uniform loading:

$$\sigma_{nom} = P/A$$

where:

P = total load applied

A = D = net width of the bar

Case 4, bending moment

$$\sigma_{nom} = Mc/I$$

where:

M = total moment applied

c = D/2 = distance from neutral axis

$I = D^3/12$ = moment of inertia

F. PROGRAM MAIN

The four programs: PROBLM (establishes SEL node coordinates based on parameters), PLIMEG (discretises the structure and computes boundary conditons), PLISOP (solves the

stress problem) and ANS (calculates the stress concentration factor) are joined together by MAIN. All card output, except the single card from subroutine ANS, is suppressed. Input data and intermediate results are passed within the program in common blocks and via computer direct-access devices. The functions of the main program are to read and store the input data, call the various subroutines in turn, provide output of the solution cards and prints, and to increment the problem parameters so as to run through a related series of evaluations.

V. PROGRAM OPERATION

A. INPUT

The problem input data deck consists of nine cards, a typical example of which is shown in Fig. 13. Card 1 lists the problem parameters: R/W, D/W, and θ. Cards 2 and 3 are title cards which may contain any message to identify the run. Card 4 lists the mesh parameters: number of SEL's, number of nodes in each element (not SEL), problem identification number and an indicator for plotting the mesh. Cards 5 through 8, are SEL connectivity cards. Each card lists the number of increments to be used in the ξ and η directions, their respective exponential coefficients, and a listing of the nodes, appropriately ordered, defining the corners of the SEL. Card 9 lists the solution parameters: number of materials, an indicator for the plane stress solution, and a code for the numerical integration technique.

The job control language cards (JCL) needed to make the program operational, details of card format, and program options are discussed in Appendix D.

B. OUTPUT

Although the output required in this particular application is a single number (SCF), a great deal more is available. All of the input data is echo checked and the following important data passed between subroutines is printed:

```
      00.3D0    0.50D0    30.0D0
        THIS IS A SAMPLE INPUT DECK -- CARD COLUMNS ARE INDICATED ON THE NEXT CARD
12345678901234567890123456789012345678901234567890123456789012345678901234567890
     4     8     1     2
     1     8    12     1    14    17     4     1    12                1.0       -1.0
     2     1    12    14    27    30    17     1     4                1.0
     5     8     3     4    17    20     7     1    12                           -1.0
     6     1     3    17    30    33    20     1    12
                 1                 0     2
```

FIG. 13 SAMPLE INPUT DECK

SEL node coordinates.

Element node coordinates with the load boundary conditions as imposed.

Element connectivity.

Displacement boundary conditions.

Material properties.

Displacements u and v for each node.

Reactions at the nodes where displacement boundary conditions were imposed.

A force equilibrium check which is the result of summing all x-direction and all y-direction reactions.

Average stress components at each node.

Average strain components at each node.

Stress concentration factor.

Although the stress concentration factor is independent of Young's modulus and Poisson's ratio, the values 30×10^6 and 0.3 respectively were coded into the program as representative of common engineering materials. The average stress and strain components are computed and printed in two ways: the cartesian coordinate components (σ_x, σ_y, and τ_{xy}) and the principal values (σ_1, σ_2, and τ_{max}) with the inclination of the principal axes (ϕ).

Other output in the form of data cards and the plotted mesh has already been mentioned.

VI. PROGRAM OUTPUT ANALYSIS

A. EQUILIBRIUM

Static equilibrium is maintained in the finite element solution of all problems studied. As indicated earlier, the equilibrium of forces is quickly checked from program output by comparing the imposed loading with the reaction summations printed by PLISOP as "equilibrium check".

The check on moment summation is slightly less convenient. Program output, including loads and reactions at all nodes, and the coordinates of nodes, is available but the actual arithmetic must be done by hand. These external calculations were made for several representative problems. The results were as required: moments and forces sum to zero.

B. MAXIMUM STRESS LOCATION

Using the program output, the location of the maximum principal stress was determined for several parameter combinations. In every case the maximum occurred tangent to the edge at the bottom of the notch.

C. CONVERGENCE

To investigate convergence behavior, we first identify the rows and columns of the SEL divisions used. The columns are referred to as band A and band B; the rows as band C and band D (Fig. 14). In what follows, band D always consists of one row of elements. This limit was imposed because

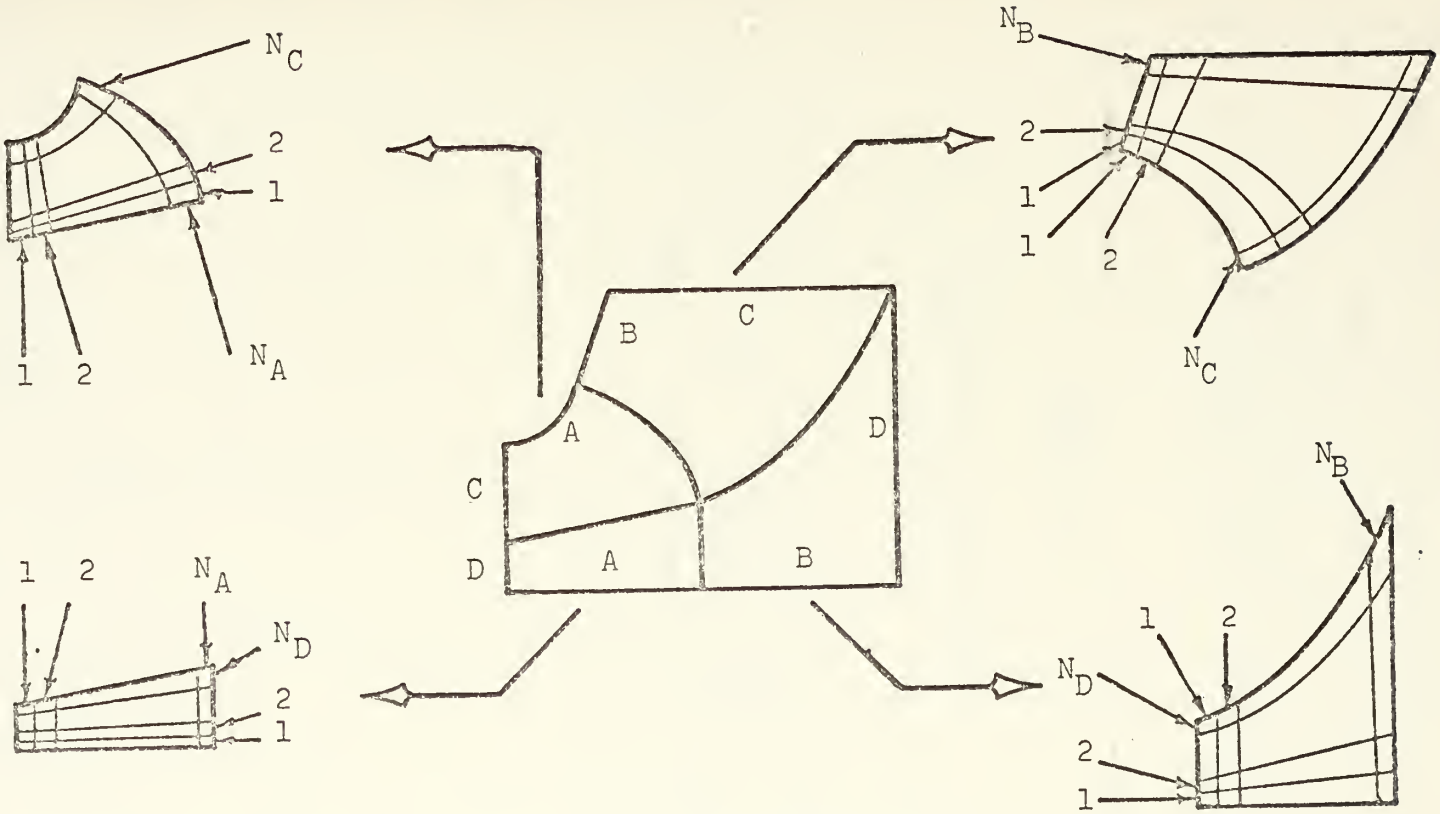


Fig. 14. SEL bands for convergence study (Numbers 1,2,...,N_A, etc., indicate band numbering.

of core storage availability in the computer used.

The refinements were accomplished by first assigning a base number to each of the other bands, then multiplying by a successively larger integer for each new mesh. Table VI shows the number of elements per band and total elements in the mesh. The mesh ultimately used is included for comparison.

TABLE VI

Convergence Subdivisions

$N_{total} = (N_A+N_B)(N_C+N_D)$ (Refer to Fig. 14)

Band Division	First Run (1)	Second Run (2)	Third Run (2)	Fourth Run (2)	Final Data Runs (3)
N_A	3	6	9	12	12
N_B	2	4	6	8	3
N_C	2	4	6	8	8
N_D	1	1	1	1	1
Total Elements	15	50	105	180	135

(1) This division was used primarily to verify correct performance of program.
(2) These divisions were used to verify and evaluate convergence.
(3) This division was used for volume data generation as an optimal compromise giving good accuracy and satifactory computer turnaround.

The refinement was done for several combinations of the problem parameters. Figure 15 is a representative plot of the stress concentration factor vs. (reciprocal of) total elements in the mesh for two combinations of the parameters as shown. In both cases only the three finest meshes used in the convergence study (50, 105 and 180 elements)are shown. Fitting a straight line through the points and extrapolating back to the axis of ordinates, an approximation for the converged result can be obtained. The value obtained using 135 elements (see below) has been added. Note that it plots on this same straight line.

Consider the upper curve shown which represents the case of a shallow notch. The extrapolated value for stress concentration factor is SCF = 1.773. The 180 element result is less than 0.8% high while the 135 element mesh used is less than 1% high. Results from studies using other parameter combinations, including $\theta \neq 0$, showed the same convergence pattern.

The mesh of 135 elements was chosen for data runs because that size mesh requires approximately 3/4 of the computer core storage space available. From a practical standpoint data collection proceeded without undue special arrangements in the computer center. The larger number of elements required elaborate special consideration by operating personnel which was not warranted by the 0.2% difference in the results.

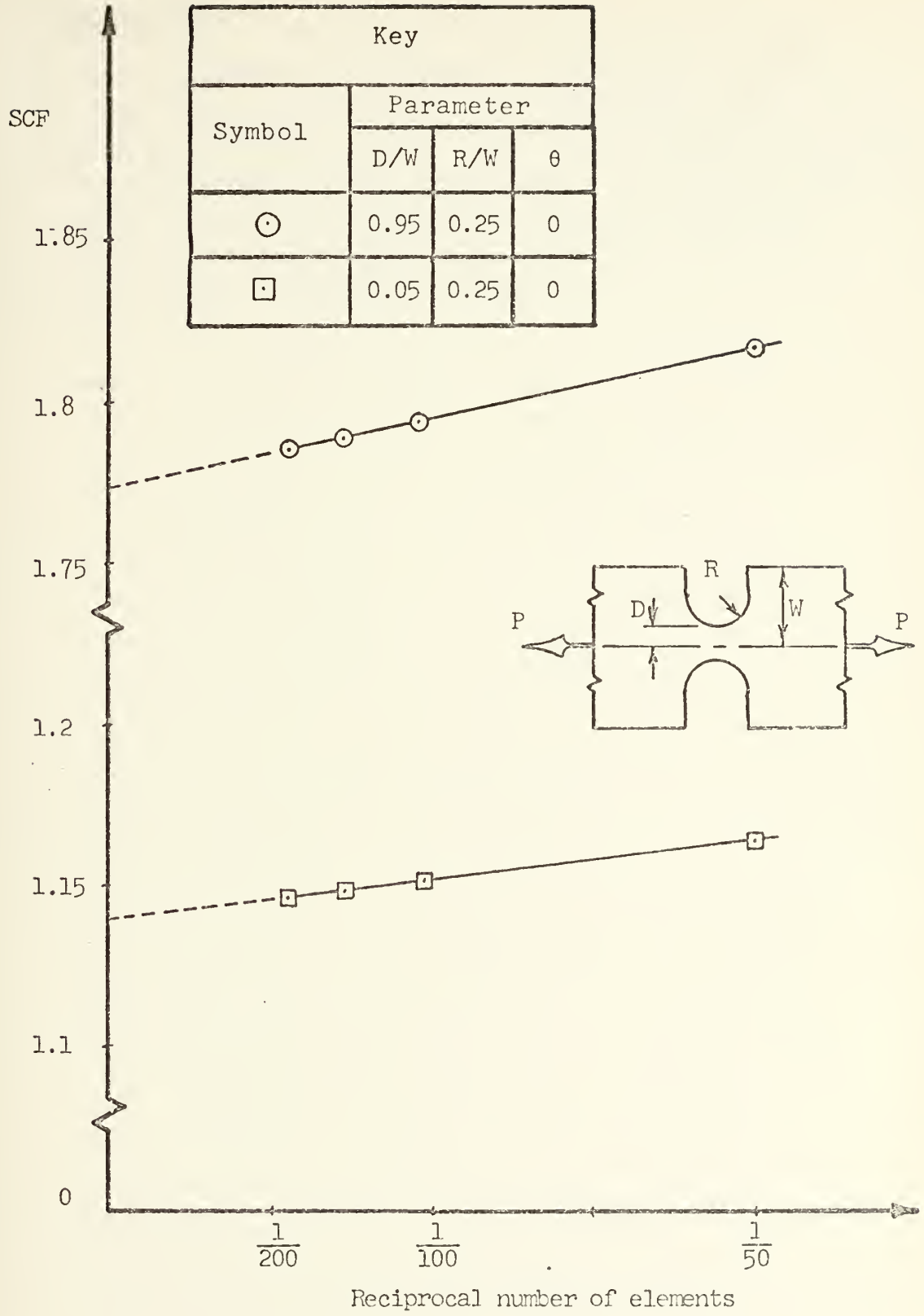


Fig. 15. Convergence Plot

D. CORRELATIONS

The data collected is discussed in the next section. As will be seen, experimental data from the literature correlates very well with values of SCF calculated. The trends established are verified and the experimental values are sometimes identical to three significant figures.

VII. DISCUSSION OF RESULTS

A. RESULTS PRESENTED

The SCF's presented in Appendix A document a representative variation of the parameters for problem 1. It is obvious that either $\theta = 90°$ or $D/W = 1$ constitutes a bar with no notch and results in stress concentration factor = 1. Several computer runs with D/W very small showed that as D/W approaches zero, SCF indeed did approach unity.

To establish the curves, $D/W = 0.1$ was taken as the initial value and incremented by adding 0.2 through $D/W = 0.9$. Likewise, R/W was incremented by steps of 0.05 from 0.1 through 0.5. θ was incremented by steps of 15° from 0° through 60°.

The data are presented as several families of curves. Each family has a specified θ associated with it. The curves are plotted as SCF vs. D/W with R/W as a parameter for each family. Linear interpolation between the curves may be used without appreciable error. Interpolation between the families for $\theta = 60°$ and $\theta = 90°$, where stress concentration factor is one, should not be attempted. Data points actually calculated are indicated by a cross. The curves through these points were generated by fitting a sixth order polynomial through them.

Only one run was made to prove the programing for problem 2; no data for this case are presented. One run was also made for each of problem 3 and 4. The data collected are presented

in Section VII D in the form of a correlation with experimental data from the literature.

Time was not available for more extensive data collection. However, now that the software has been perfected and proved, it is a perfectly straightforward task to obtain additional data as computer time becomes available. Appendix D suggests the parameter ranges which should be investigated.

B. EFFECT OF NOTCH DEPTH

Selected data from the studies of problem 1 are plotted in Fig. 16. The parameter D/R is constant. Experimental data plotted is from Flynn and Roll; however Kikukawa's data shows the same trends. The data in the experiments was for the symmetrically located U-shaped notches as in problem 1, with $\theta = 0$. Both Neuber's results and Flynn and Roll's experimental data are included for the hyperbolic shaped symmetric notches with the same parameters.

The general close agreement between the FEM and experimental values for the U-shaped notch is easily seen. The largest difference noted is for $W/D = 4.33$ where the difference is less than 2%.

We might examine the difference by utilizing an aspect ratio defined as effective length from the notch to point of load application divided by notch depth. Table VII compares this value. As can be seen, the calculations allow a much more favorable (i.e., larger, and thus more realistic) ratio than that provided in the experiment.

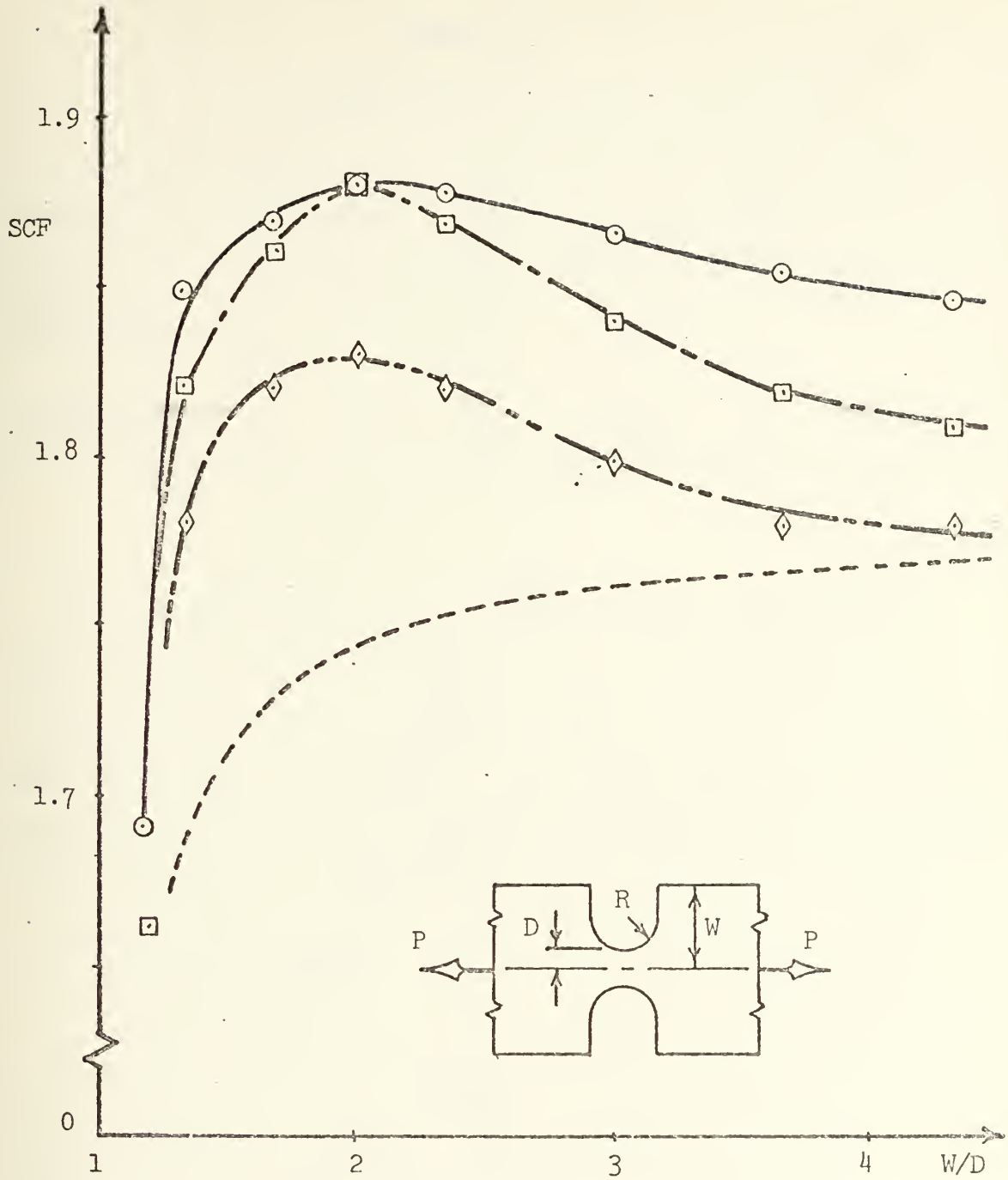


Fig. 16. Effect of notch depth on SCF for R/W = 1.5

Numerical U-notch (Finite Element)

Experimental U-notch (Flynn & Roll)

Experimental Hyperbolic notch (Flynn & Roll)

Analytical Hyperbolic notch (Neuber)

TABLE VII

Aspect Ratio L/(W - D)

Source	W/D = 2	W/D = 3	W/D = 4.33
FEM Calculations	10	7.5	6.5
Experiments	8.6	4.33	2.6

The maximum point on the curve in Fig. 16 occurs at W/D = 2, a relatively shallow notch. Experimental data matches at this point exactly. Note, the error that would be made if Neuber's hyperbolic notch result was used for such a notch is more than 7%. Experimental data presented by Flynn and Roll point to a systematic error in Neuber's results.

C. EFFECT OF ANGLE θ

The data plotted in Fig. 17 compares the stress concentration factor for θ = 60° (SCF_{60}) to the stress concentration factor for θ = 0° (SCF_0). The parameter used is W/D. Two experimentally determined curves show that increasing the angle of the V from 0° to 60° decreases the SCF_{60}. This effect is more profound as the W/D ratio is increased (i.e., for deeper notches).

Replotting the data as in Fig. 18 shows the variation of SCF_{60} with changing W/D for constant SCF_0. Note the close correlation of the experimental data and FEM data.

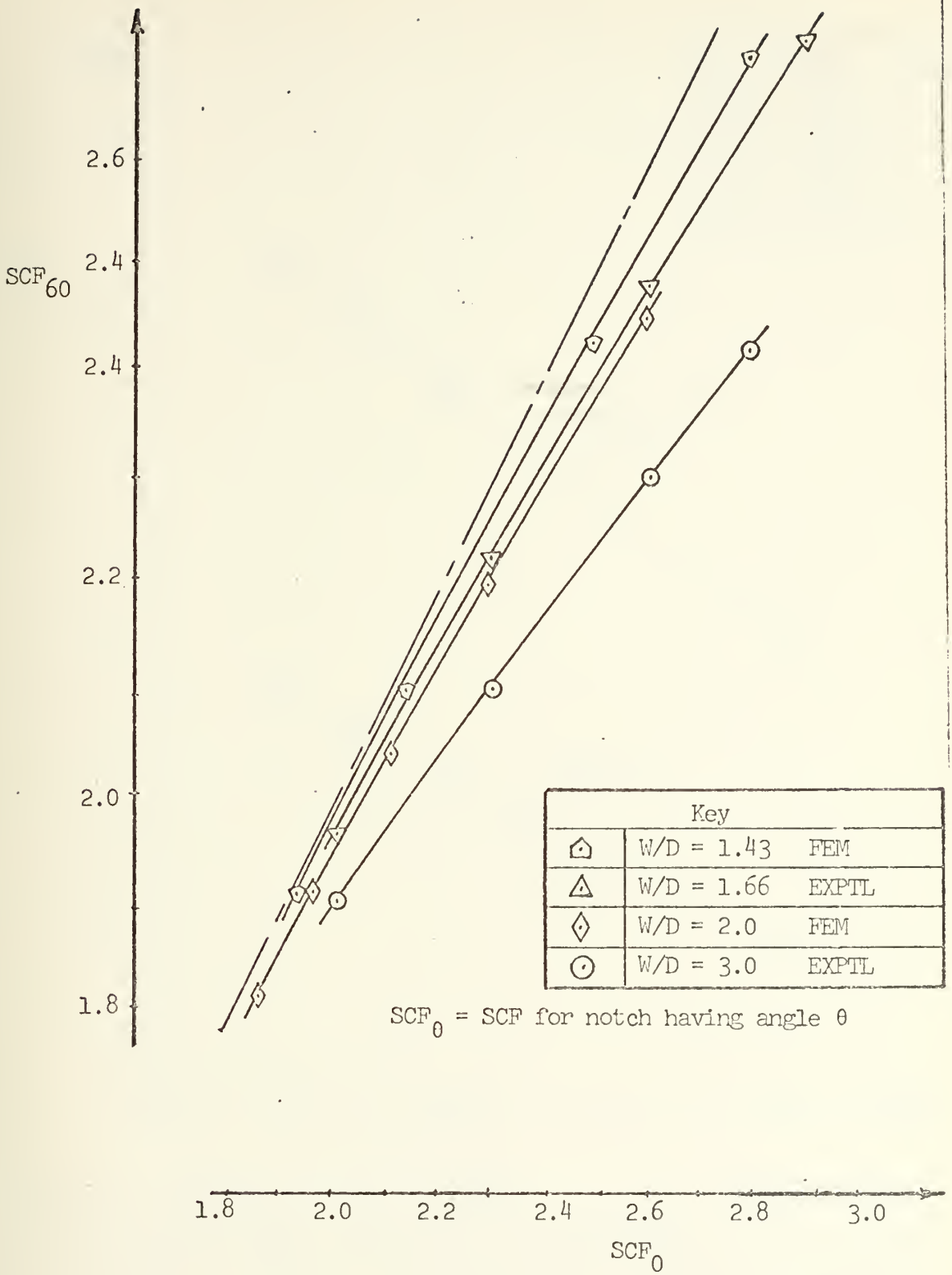


Fig. 17. Effect of V angle on SCF. The dot-dash line ($SCF_{60} = SCF_0$) is included for reference to emphasize that SCF_{60} is always less than SCF_0 for any value of the other parameters. Similar data could be plotted for other values of θ.

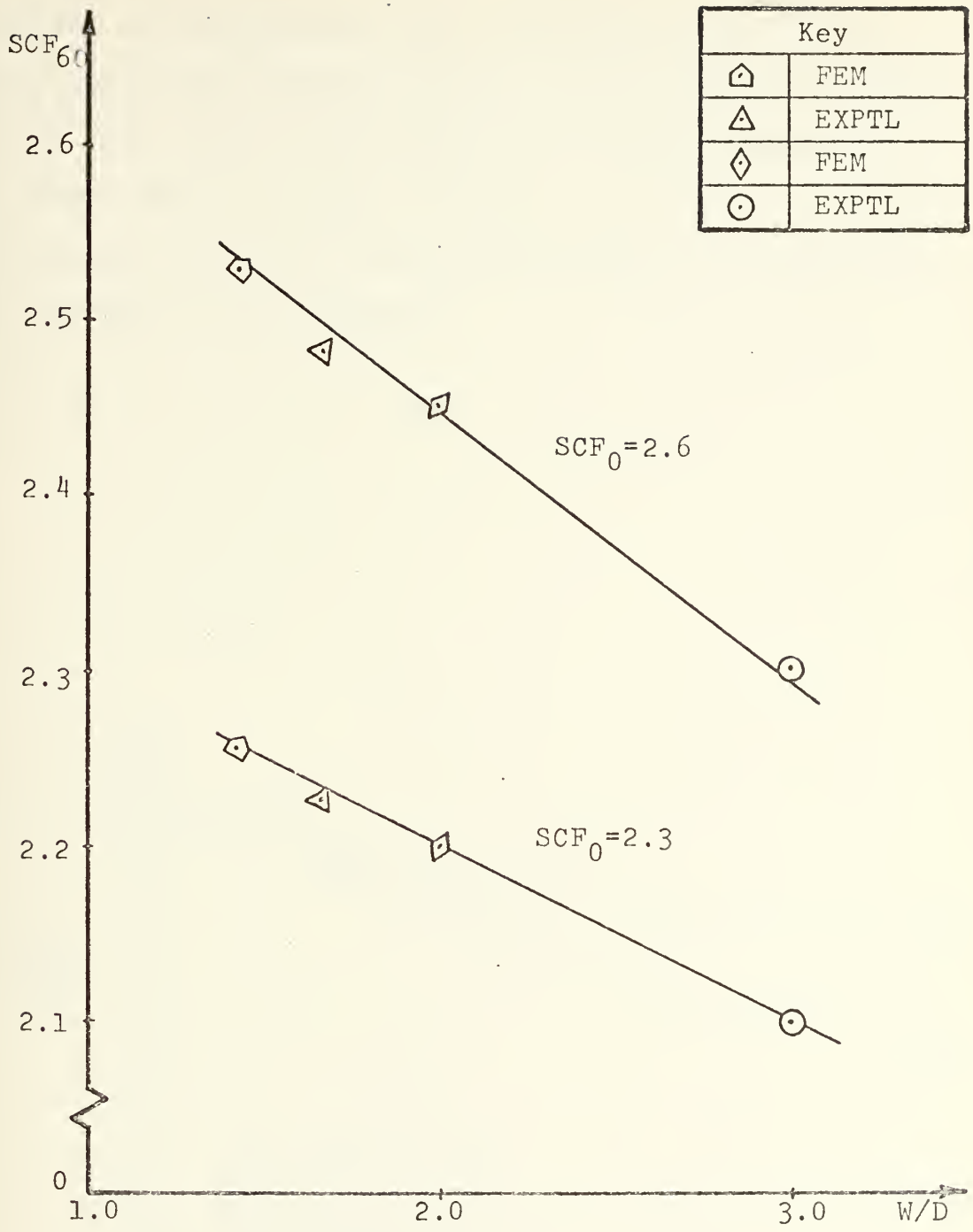


Fig. 18. Effect of W/D on SCF_{60} These lines represent profiles (for $SCF_0 = 2.3$ and $SCF_0 = 2.6$) of the data shown in Fig. 17. The same plot symbols are used as in Fig. 17.

Figure 19 demonstrates that increasing the angle θ reduces the stress intensification and this effect is more profound for smaller radius notches than for notches where the bottom radius is large. That is, the SCF for a given notch may be reduced by increasing either R or θ while the other parameters are unchanged.

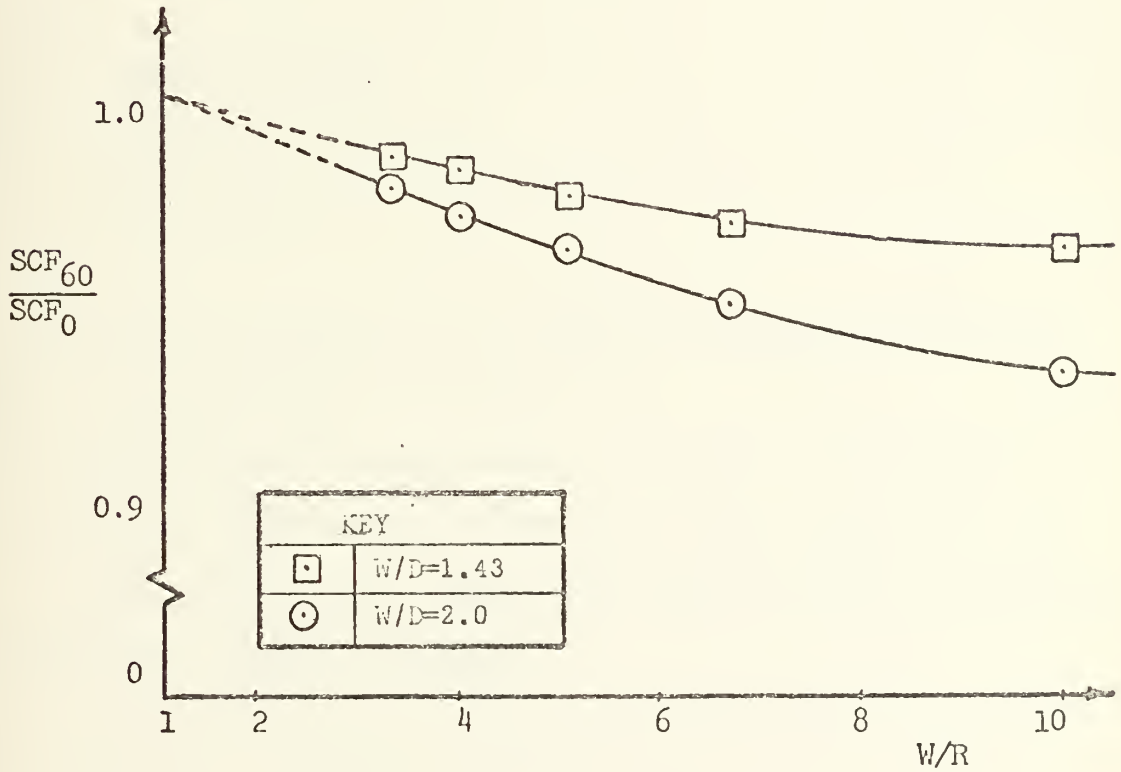


Fig. 19. Effect of angle θ on SCF as function of radius

D. SINGLE NOTCH GEOMETRY

The only experimental data available dealing with the single notch geometry is that presented by Cole and Brown [5]. Their data is for a particular case of loading such that the line of action of the applied force is through the midpoint of the net width of the bar. By an appropriate combination of the SCF's calculated for problems 3 and 4 this loading can be reproduced as shown in Fig. 20. The defining relationship for SCF as used by Cole and Brown is:

$$SCF_* = \frac{\sigma_{max}}{P/D}$$

where

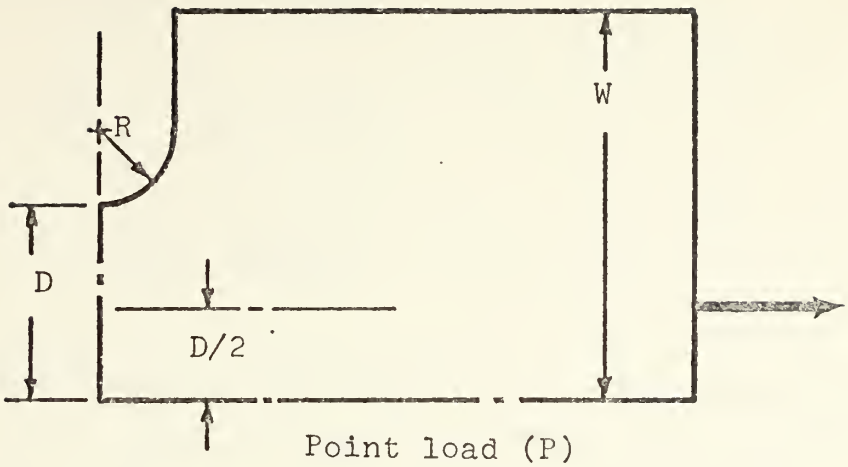
σ_{max} = maximum stress

P = total load applied

D = net width

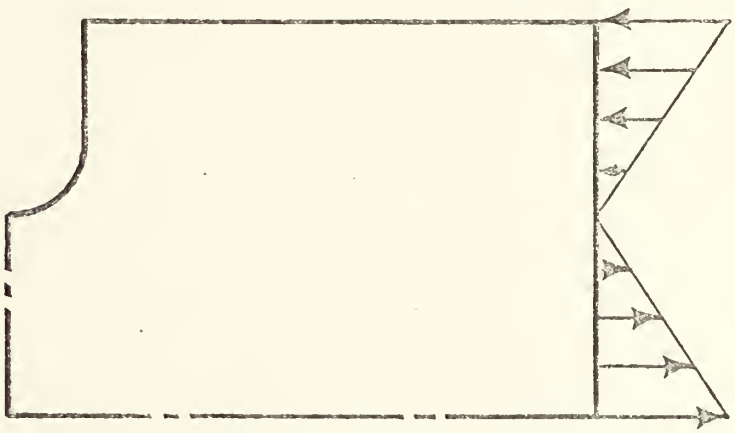
The defining relationships for problem 3 (SCF_3) and problem 4 (SCF_4) are contained in Section IV E.

For the parameter values D/W = 0.5, R/W = 0.1 and $\theta = 0°$, Cole and Brown report $SCF_* = 1.95$. The computed results are $SCF_3 = 7.85$, $SCF_4 = 1.98$. Making the appropriate substitutions and solving for maximum stress:



Point load (P)

Uniform load ($\sigma = P/W$)



Bending moment [$M = (W - D)P/2$]

Fig. 20. Bar with one notch only. A point load decomposed into constant and linearly varying components. St. Venant's principle accounts for the absence of higher order components.

$$\sigma_{max} = SCF_3\left(\frac{P}{D}\right) - SCF_4\left(\frac{Mc}{I}\right)$$

$$= (SCF_3 - 3\ SCF_4)\left(\frac{P}{D}\right)$$

$$= 1.91\left(\frac{P}{D}\right) = SCF\left(\frac{P}{D}\right)$$

Thus the FEM analysis gives SCF = 1.91 compared to Cole and Brown's 1.95. There is approximately 2% difference in the results. A special computer run with this specific loading gave SCF = 1.91 also. This calculation served to verify superposability of the two cases and the correctness and internal consistency of the programming and the inputs.

VIII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The FEM program as presented in this thesis can be used to calculate the stress concentration factor for the specific problems listed, over a wide range of the parameters used. Further, by appropriate superposing of these four specific problems, the SCF can be calculated for bars which are notched either on one side only or symmetrically on both sides and are subjected to general tensile and bending loads.

One of the conclusions from an analysis of the data obtained for problem 1, the case of the symmetrically notched bar subjected to a tensile load only, and a comparison with tabulations and recommendations in the literature is that the three parameters (minimum radius of curvature, gross width of the bar, and net width of the bar) do a rather good job of characterizing the geometry of the notch, in the sense that interpolated values of the SCF obtained from available data agree rather well with the more accurate values obtained by FEM analysis in this study.

However, the analysis also shows that there are some small but systematic deviations that are assignable to the fact that these three parameters are not sufficient to characterize the geometry of the notch completely. The data presented in Appendix A accordingly provides a more complete and accurate representation of SCF's in this case than was previously available.

The stress concentration factors presented in the literature for several specific cases of U-shaped and V-shaped notches are essentially correct. However, the data presented is limited, mainly because of the difficulties associated with experimental investigations. This program overcomes those difficulties and can solve for a single SCF in less than three minutes of computer time as compared to several hours by accepted experimental methods.

B. RECOMMENDATIONS

Accordingly, it is recommended that the program described herein be used to determine SCF's for problems 1 through 4 and that the results be published for general use. Appendix D presents a specific plan for obtaining the necessary results.

Further, if interest in other notch shapes warrants it, this program should be appropriately modified and the SCF's calculated and published.

An axi-symmetric stress analysis program has been written by Bahkshandehpour [3] as part of his NPS thesis. That program could be similarly adapted to calculate SCF's for general axisymmetric geometries having notches.

APPENDIX A

CURVES OF RESULTS

The stress concentrations presented in this appendix are for the problem of symmetrically located, round bottom, V-shaped notches subjected to uniform tensile loading. The data collected is plotted as crosses. A sixth order polynomial is shown fitted through these points.

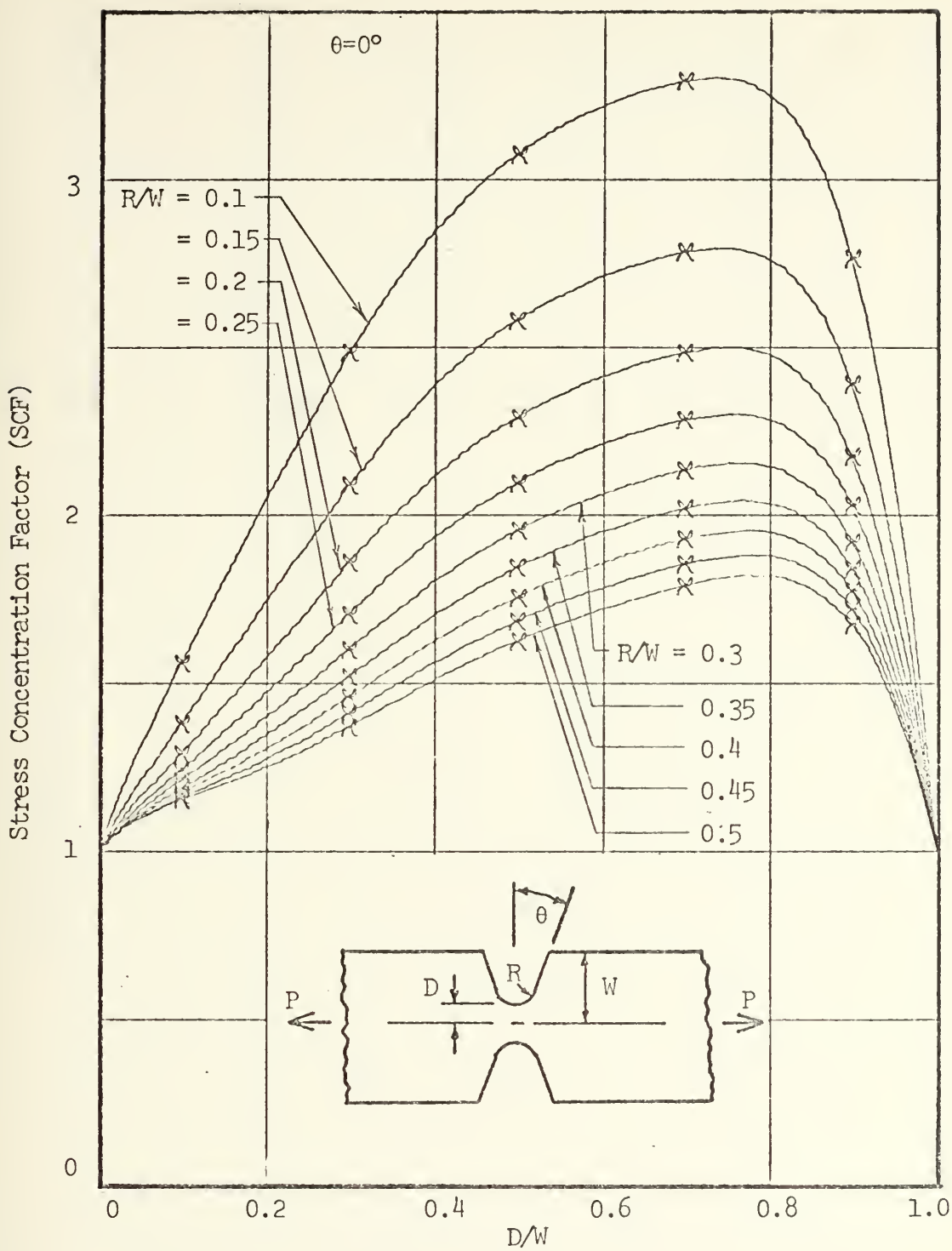
The curves are plotted as SCF vs. D/W using R/W as a parameter with each family representing a single θ value, where

R = radius of the bottom of the notch

D = net half width of the bar

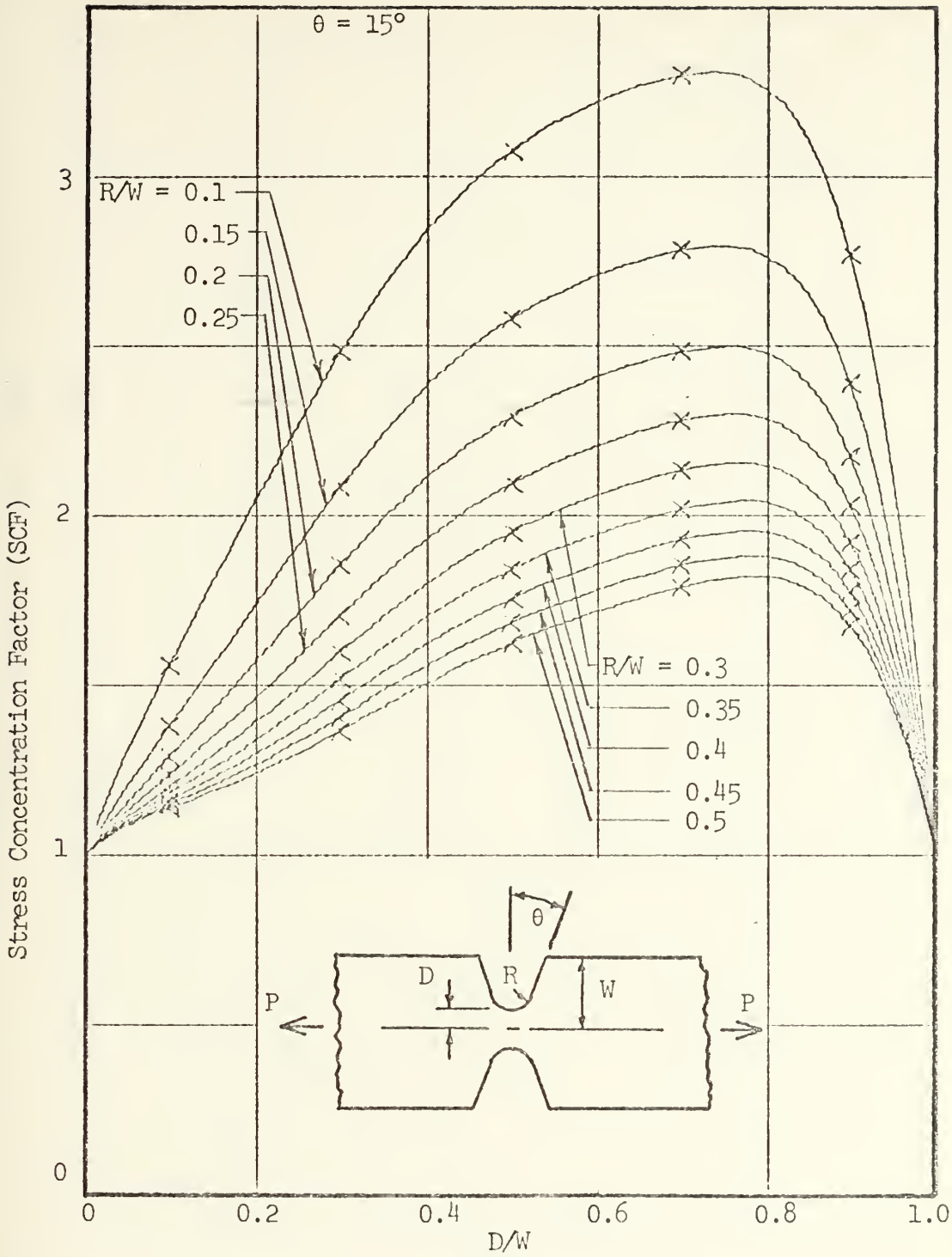
W = gross half width of the bar

θ = half included angle of the V



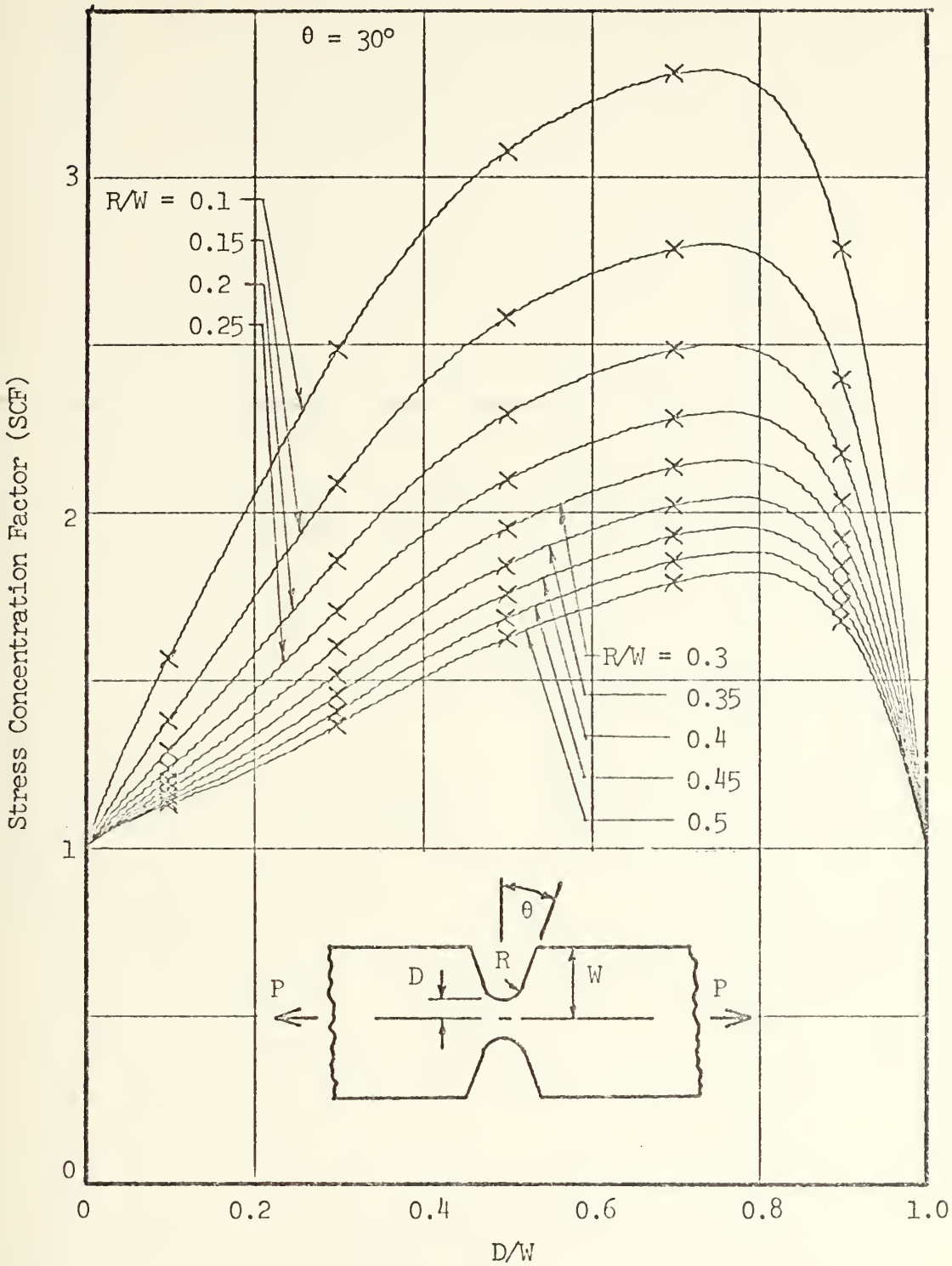
Plane stress values of SCF for $\theta = 0°$.

x indicates FEM data. $\sigma_{max} = SCF(\frac{P}{2D})$



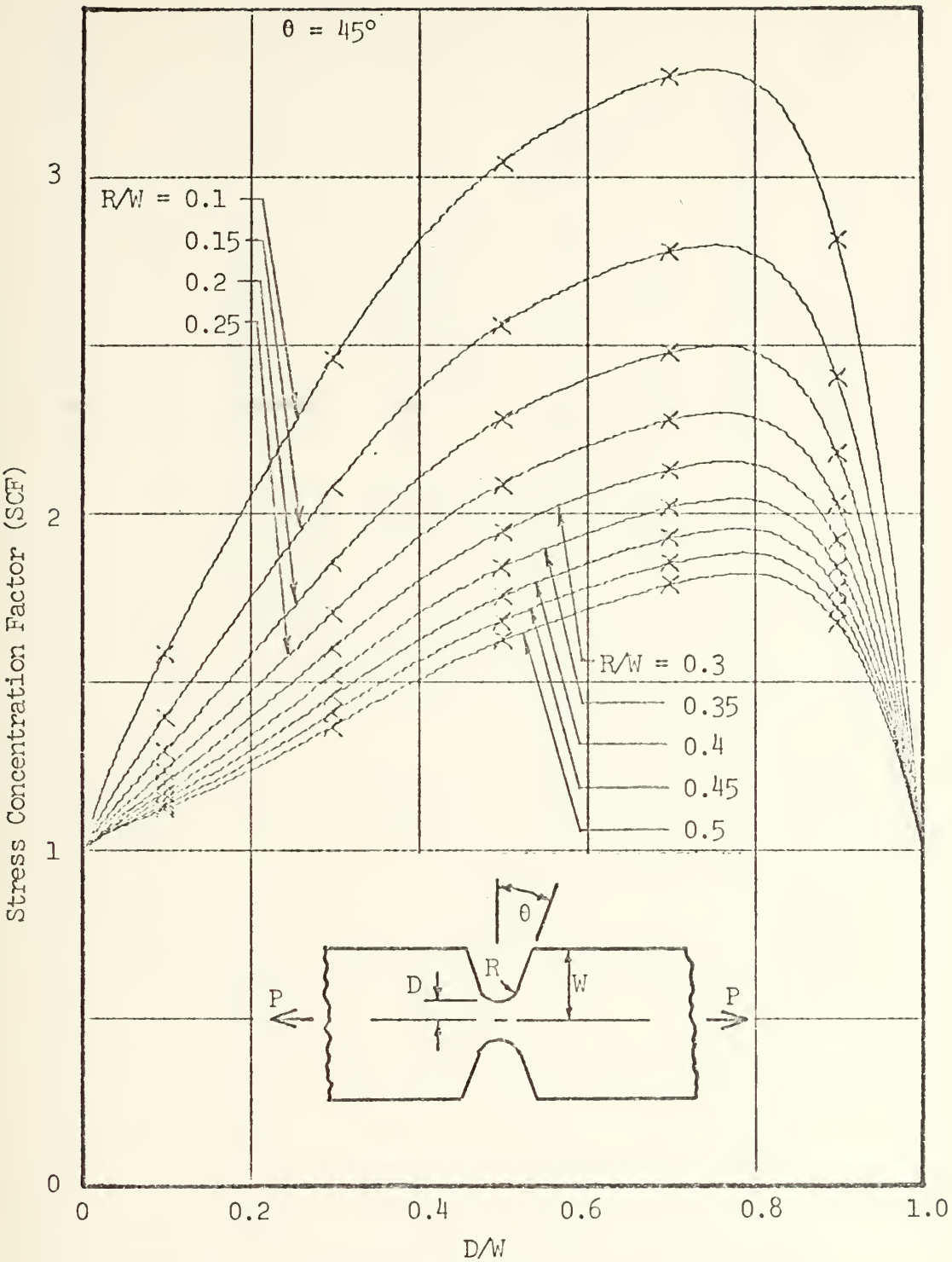
Plane stress values of SCF for θ = 15°.

x indicates FEM data. $\sigma_{max} = SCF(\frac{P}{2D})$



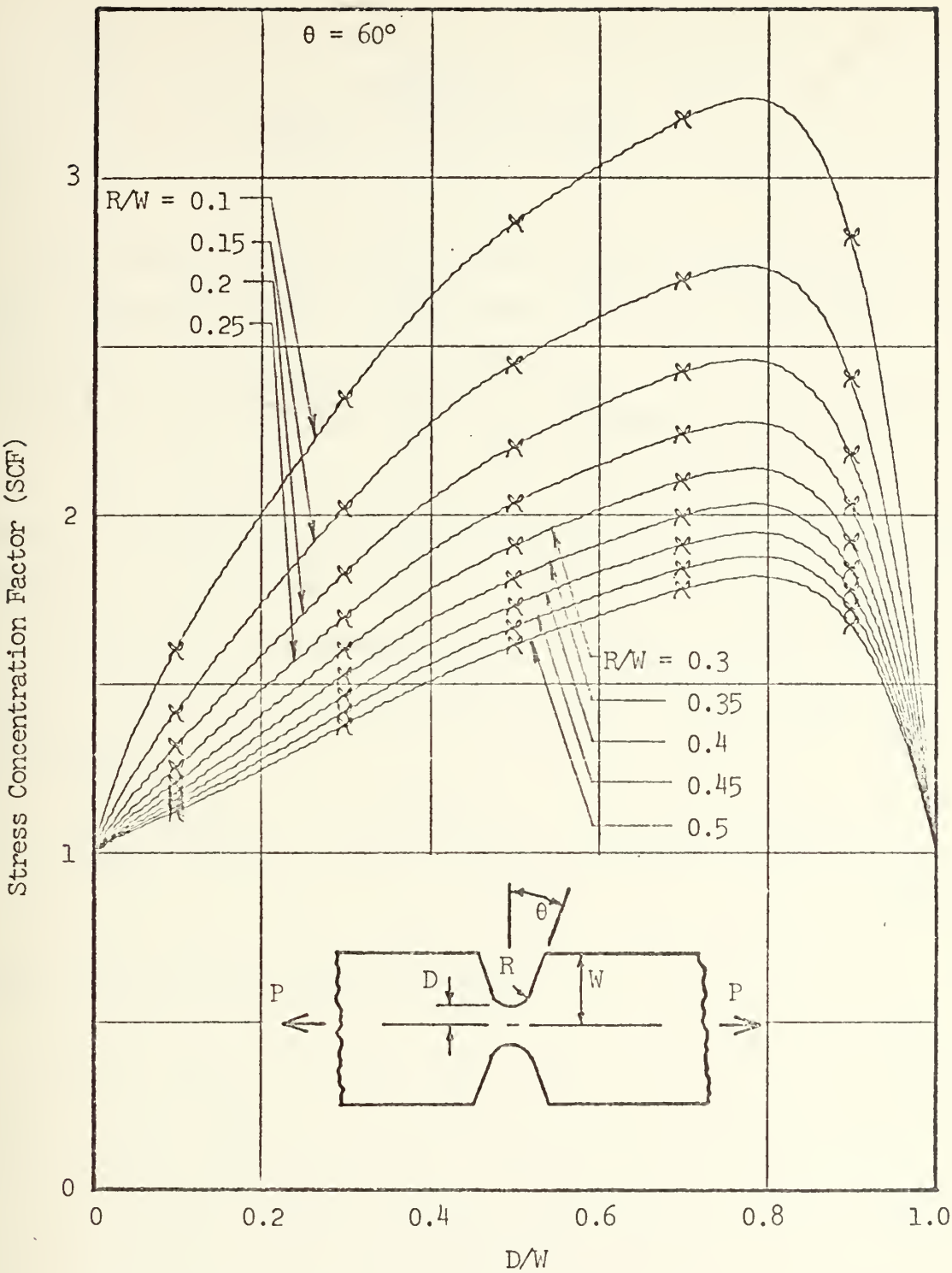
Plane stress values of SCF for θ = 30°

x indicates FEM data. $\sigma_{max} = SCF(\frac{P}{2D})$



Plane stress values of SCF for $\theta = 45°$.

x indicates FEM data. $\sigma_{max} = SCF(\frac{P}{2D})$



Plane stress values of SCF for $\theta = 60°$.

x indicates FEM data. $\sigma_{max} = SCF(\frac{P}{2D})$

APPENDIX B

NEUBER'S NOTCH ANALYSIS

This appendix is included since we have been unable to find a reference which quotes these useful and important results in a brief and clear fashion.

We are concerned with the two-dimensional case of an axial load transmitted in the y-direction through the net section of a thin bar (plane stress case) having a hyperbolic boundary. All quantities which follow are for unit thickness.

To describe the geometry and to provide a basis for the solution for elastic stress distribution, Neuber employed the mapping

$$z = c \cosh \zeta, \quad \zeta = \cosh^{-1}(z/c) = \ln[(z/c)+(z^2/c^2 - 1)^{\frac{1}{2}}] \tag{B1a,b}$$

where z and ζ are complex variables

$$z = x + iy \quad , \quad \zeta = \xi + i\eta \tag{B2a,b}$$

and i is the imaginary unit. In real terms this results in

$$x = c \cosh \xi \cos \eta \; ; \quad y = c \sinh \xi \sin \eta \tag{B3a,b}$$

$$\xi = \tfrac{1}{2} \cosh^{-1}(G+H) \quad , \quad \eta = \tfrac{1}{2} \cos^{-1}[G-H] \tag{B4a,b}$$

where

$$G = (x^2+y^2)/c^2 \quad ; \quad H^2 = G^2 + 2(y^2-x^2)/c^2 + 1 \tag{B5a,b}$$

This mapping provides a correspondence between the xy cartesian system of coordinates and a $\xi\eta$ system useful for defining the geometry of the notch. Lines of constant ξ are ellipses

$$\left(\frac{x}{c \cosh \xi}\right)^2 + \left(\frac{y}{c \sinh \xi}\right)^2 = 1 \qquad \text{(B6)}$$

while lines of constant η are the hyperbolas

$$\left(\frac{x}{c \cos \eta}\right)^2 - \left(\frac{y}{c \sin \eta}\right)^2 = 1 \qquad \text{(B7)}$$

Figure B-1 shows the pattern of the $\xi\eta$ network.

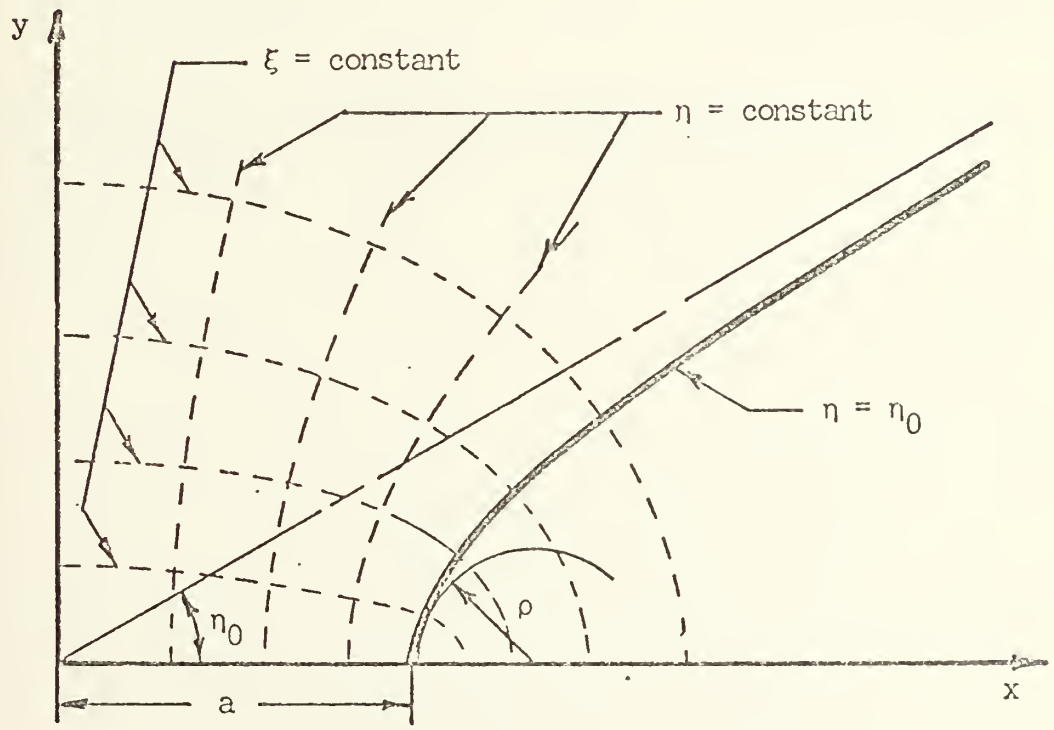


Fig. B1. Neuber's hyperbolic, symmetric notches in an infinite plate.

A complex Airy stress function is used to solve the elasticity problem so that

$$\sigma_\eta \equiv \tau_{\xi\eta} \equiv 0 \tag{B8}$$

along the notch boundary which is the hyperbola

$$x^2 - y^2 \cot^2\eta_0 = c^2 \cos^2\eta_0 \tag{B9}$$

corresponding to the value $\eta = \eta_0$. The half width at the base of the notch is the value of x when y = 0. We denote this by a, so that we are able to evaluate

$$c = a \sec \eta_0 \quad . \tag{B10}$$

This choice of c selects a definite shape, namely that which has net width 2a and whose hyperbolic boundary has asymptotes

$$y = \pm x \tan \eta_0 \quad . \tag{B11}$$

It is significant to note that except for a change of overall scale, the value of η_0 is the single parameter which defines the geometry of the notch. The two "halves" of the hyperbolic boundary are fixed relative to each other by the fact that their mutual asymptotes intersect at the origin x = y = 0.

One of the most significant derived quantities is the minimum radius of curvature of the hyperbola, ρ, which occurs at the base of the notch.

$$\rho = a \tan^2 \eta_0 \quad . \tag{B12}$$

The solution, not given in detail here, permits finding the stress state $(\sigma_x, \sigma_y, \sigma_{xy})$ or $(\sigma_\xi, \sigma_\eta, \tau_{\xi\eta})$ at any point of the field. A constant, b, is involved in this solution. For our purposes we need only the stresses along the line $y = 0$, viz:

$$\sigma_x = \sigma_\eta = 2b \csc^3\eta \, (\sin^2\eta - \sin^2\eta_0)$$

$$\sigma_y = \sigma_\xi = 2b \csc^3\eta \, (\sin^2\eta + \sin^2\eta_0) \tag{B13,a,b,c}$$

$$\tau_{xy} = \tau_{\xi\eta} = 0$$

One of the results we will use in Appendix C is the sum

$$\sigma_x + \sigma_y = 4b \csc \eta \tag{B14}$$

along $y = 0$.

To determine the constant b, we note that the tensile force (per unit thickness) transmitted in the y-direction is

$$P = 2 \int_0^a \sigma_y \, dx \tag{B15}$$

Substituting Eq. (B13b) and solving for b, we get

$$b = (P \cos \eta_0)/2a(\pi - 2\eta_0 + \sin 2\eta_0) \tag{B16}$$

Substituting Eq. (B16) into (B13b), and evaluating at $\eta = \eta_0$, we get the maximum tensile stress (at the base of the notch) to be

$$\sigma_{max} = (2P \cot \eta_0)/a(\pi - 2\eta_0 + \sin 2\eta_0) \qquad (B17)$$

The nominal tensile stress is

$$\sigma_{nom} = \frac{P}{2a} \qquad (B18)$$

and the stress concentration factor is

$$SCF = \sigma_{max}/\sigma_{nom} = (4 \cot \eta_0)/(\pi - 2\eta_0 + \sin 2\eta_0) \qquad (B19)$$

In terms of the ratio of minimum radius of curvature ρ and net half-width a, we have

$$SCF = 4/k[\pi - 2 \arctan k + 2k/(1+k^2)] \qquad (B20)$$

where

$$k = \rho/a \qquad (B21)$$

This shows the SCF in terms of the ratio of two evidently significant and measurable geometric dimensions.

APPENDIX C

THREE DIMENSIONAL EFFECT

1. PREAMBLE

Before proceeding with this discussion, the following points should be made clear. First, there is every reason to place full reliance upon the accuracy of the determinations made by the two-dimensional FEM program, that is, to assert that these calculations accurately represent the theoretical two dimensional problem (plane stress, as presented here). From this standpoint, the study of three dimensionality is intended only to assess the valididty of photoelastic determinations which were compared to the FEM determinations, and we in no way depend upon three dimensional behavior in presenting, with great confidence, our results for the two dimensional case. However, one cannot evade the responsibility of making some sort of evaluation of the effect of finite thickness upon determinations valid, strictly speaking, only for cases of zero thickness or infinite thickenss.

2. A SPECIFIC PROBLEM

In addition to other difficulties of photoelastic evaluations, there is an inherent error which is due to the fact that all photoelastic models have finite thickness so that the conditions of plane stress are violated, at least to some extent.

To a first approximation, as shown by Timoshenko and Goodier [15], the variation of a two-dimensional stress field, due to finite thickness, may be obtained by employing, instead of the usual Airy stress function $\phi_0(x,y)$, satisfying

$$\nabla^4\phi_0 \equiv 0 \quad , \quad \frac{\partial^2\phi_0}{\partial x^2} = \sigma_y \quad , \quad \frac{\partial^2\phi_0}{\partial y^2} = \sigma_x \quad , \quad \frac{\partial^2\phi_0}{\partial x\partial y} = -\tau_{xy} \qquad \text{(C1a,b,c,d)}$$

a modified function

$$\phi(x,y;z) = \phi_0(x,y) - \frac{\nu z^2}{2(1+\nu)}\ \theta_0(x,y) \qquad \text{(C2)}$$

where

$$\theta_0(x,y) = \nabla^2\phi_0 = \frac{\partial^2\phi_0}{\partial x^2} + \frac{\partial^2\phi_0}{\partial y^2} \qquad \text{(C3)}$$

The (modified) stresses

$$\sigma_x = \frac{\partial^2\phi}{\partial y^2} \quad ; \quad \sigma_y = \frac{\partial^2\phi}{\partial x^2} \quad ; \quad \tau_{xy} = -\frac{\partial^2\phi}{\partial x\partial y} \qquad \text{(C4a,b,c)}$$

obtained from this modified stress function, thus have a parabolic dependence upon the third dimension, and, in general,

agree with the elementary (strictly two-dimensional) solution only on the plane of symmetry, $z \equiv 0$.

Note that θ_0 is the first tensor invariant of the elementary solution, that is

$$\theta_0 = (\sigma_x)_0 + (\sigma_y)_0 \qquad (C5)$$

where we have added the subscript 0 to indicate that these are the elementary or unmodified stresses.

In photoelastic experiments, the light beam is affected by the distance-average stress-state and this differs somewhat from the stress-state at mid thickness. It is the latter, however, which corresponds to the theoretical plane stress distribution which the photoelastic evaluation is intended to reveal.

To determine the correction it is necessary to have, locally, second derivatives of the first tensor invariant. This involves two-fold differentiation of a stress field which itself has been determined, usually, by graphical or numerical integration procedures. The problem is somewhat easier to deal with at points on the free edge. Here there is only one stress, namely that in a direction parallel to the free edge, with which we are concerned, and all that is required is two-fold differentiation of θ_0 in a direction normal to the edge. Nevertheless, it is quite unlikely that sufficiently accurate stress determinations will have been made on the interior of the model area to permit a reliable evaluation of the second derivative needed.

In the particular case with which we are concerned in this thesis, namely the matter of stress concentrations at the bottom of notches, we can construct a plausible theoretical means of obtaining the desired information. This procedure, and experimentally accomplished equivalent procedures are, however, of doubtful accuracy and probably do no more than give a very rough estimate of the proper correction which, fortunately, usually is not large.

The problem, specifically, is to determine $\frac{d^2\theta_0}{dn^2}$

where θ_0 is the sum of principal stresses and n is the direction normal to the boundary at the point at which the correction is to be made. The evaluation is at the boundary.

3. PROCEDURES TO SOLVE THE PROBLEM

The following procedures suggest themselves. First, one can use the photoelastic stress information, properly integrated so as to obtain the stress-sum, and perform two-fold differentiation. Or, still proceeding experimentally, one can perform another experiment, using an interferometric procedure, to get the isopachic lines, which provide a direct experimental evaluation of the (average) stress-sum, and then differentiate the field twice in the normal direction. Third, one can analytically determine the desired second derivatives by studying an available theoretically exact solution to a geometry which "matches" that of the case under investigation. Fourth, one can construct a photoelastic model of such a geometry for which an analytic solution is available and infer

the desired correction by comparing photoelastic results with the theoretical results and then apply the same correction to the case under investigation.

Although superficially these procedures seem reasonable, each has flaws of fatal or near-fatal magnitude. The first two methods described do indeed relate to the geometry of the case at hand rather than to some substitute geometry. Thus, in theory, they are capable of providing the desired information. However, the fact that a stress-sum field must be constructed with sufficient accuracy that a second derivative can be relied upon indicates the need for a degree of precision considerably beyond what might be regarded as normally achievable.

The third and fourth methods are essentially the same — one is dealing with a substitute geometry for which a theoretical result is available. In the third method one needs a more detailed mathematical representation of the solution; in the fourth method all that is required is the theoretical value of the edge stress — and a second photoelastic determination. In each of these methods however, the reliance is upon the assumption that the second derivative of the stress-sum is the same for the substitute geometry as for the case of interest, or at least that the ratio of this second derivative to the edge stress itself is the same for the two cases.

One is quite disinclined to believe that there is any real justification for such reliance. If the substitute geometry

(with known analytic solution) is sufficiently close to the actual geometry that one may rely upon the coincidence of the second derivative of the stress sum, then indeed there should be even closer coincidence between the edge stresses themselves so that a photoelastic investigation is not called for at all. It is the very fact that the geometry of interest does not coincide with a geometry for which there is an available analytic solution that causes us to make the photelastic study in the first place.

However, even though we may have deep and quite reasonable doubts concerning the accuracy of our evaluation or estimate of the second derivative of the stress-sum (or its equivalence in other terms), nevertheless, for thin photoelastic models there is reason to believe that the corresponding variation in edge stress through the thickness of the model is "small" so that a greater degree of error may be tolerated in this estimate than would be tolerable in the basic evaluation of the edge stress.

Our own study of this matter has involved (1) analytical differentiation of the stress-sum field for a known geometry "approximating" that of the case under study, (2) numerical differentiation of the stress-sum field obtained by use of a two-dimensional FEM program, and (3) actual use of a three-dimensional FEM program.

In the following paragraphs we concern ourselves with a U-shaped notch. The radius of the bottom of the notch is R = 0.2 inch, the gross half width of the bar is W = 1.3 inches, the net half width is D = 0.3 inch and the half thickness is 0.125 inch. The angle θ, as used in the text, is $\theta = 0°$.

a. Analytical Differentiation

1. Neuber's Deep Notch

The geometry of Neuber's hyperbolic notch in an infinite plate is shown in Fig. B1. Equations B13 and B14 in Appendix B give the following expressions for stress:

$$(\sigma_y)_0 = 2b \csc^3\eta \, (\sin^2\eta + \sin^2\eta_0) \qquad (C6)$$

$$\theta_0 = (\sigma_x + \sigma_y)_0 = 4b \csc \eta \qquad (C7)$$

Letting $\csc \eta = (1 - u^2)^{-1/2}$ and designating $\frac{d}{dn}$ by $'$, we get

$$\theta_0'' = 4b \, (1+2u^2)(1-u^2)^{-5/2} \csc^2\eta_0/D^2 \qquad (C8)$$

Along $y = 0$, σ_y varies both with respect to x and z; letting

$$c = \nu/2(1+\nu) \quad ,$$

we get from eq. (C4):

$$\sigma_y = (\sigma_y)_{2D} - c\ \theta''\ z^2 \tag{C10}$$

Taking $2\lambda = t$, the thickness, and evaluating the constant, b:

$$p = 4 \int_0^D \int_0^\lambda \sigma_y\ dz\ dx \tag{C11}$$

$$b = p \cos \eta_0 / 4\lambda D[G - 3H/2] \tag{C12}$$

where $$G = \pi - 2\eta_0 + \sin 2\eta_0 \tag{C13}$$

$$H = \lambda^2 c \cot^3 \eta_0 / D^2 \tag{C14}$$

From eq. (C10)

$$(\sigma_y)_{max} = 4b \csc \eta_0 \tag{C15}$$

$$(\sigma_y)_{edge} = (\sigma_y)_{max} - c\ \theta_0''\ \lambda^2 \tag{C16}$$

Letting

$$\Delta\sigma_y = (\sigma_y)_{max} - (\sigma_y)_{edge} \tag{C17}$$

$$\frac{\Delta\sigma_y}{(\sigma_y)_{max}} = H \csc \eta_0\ (1 + 2 \cos \eta_0) \tag{C18}$$

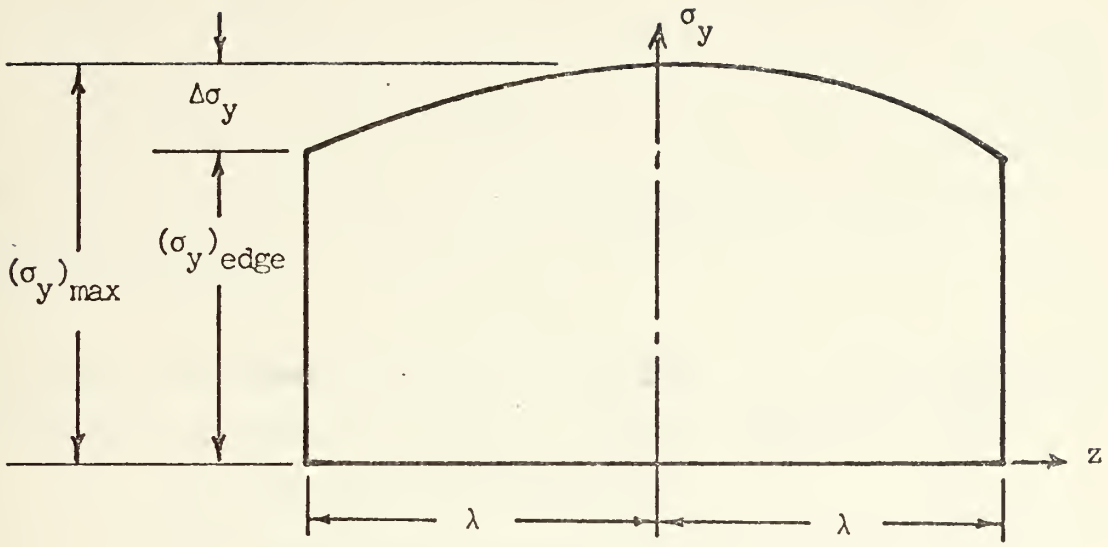


Fig. C1. Stress profile through the thickness

Figure C1 shows the stress profile through the model thickness. The photoelastician sees σ_y averaged through the thickness 2λ:

$$(\sigma_y)_{ave} = (\sigma_y)_{max} - \frac{\Delta\sigma_y}{3} \tag{C19}$$

Consider the case: $D = 0.3$, $\rho = 0.2$, $\lambda = 0.125$, $\rho = D \tan^2\eta_0$. We find

$$\overline{\theta_0}'' = 18.85$$

$$(\sigma_y)_{ave} = 0.982\ (\sigma_y)_{max} \qquad (C.20)$$

(Here and later $\overline{\theta_0}''$ denotes the evaluation of θ_0'' at the base of the notch.)

That is to say the photoelastic evaluation of stress provides an uncorrected value for $(\sigma_y)_{max}$ that is nearly 2% low.

2. Circular Hole in a Plate

One substitute geometry which suggests itself is that of a circular hole in a plate under uniaxial tension. Figure C2 shows a round hole in a large plate.

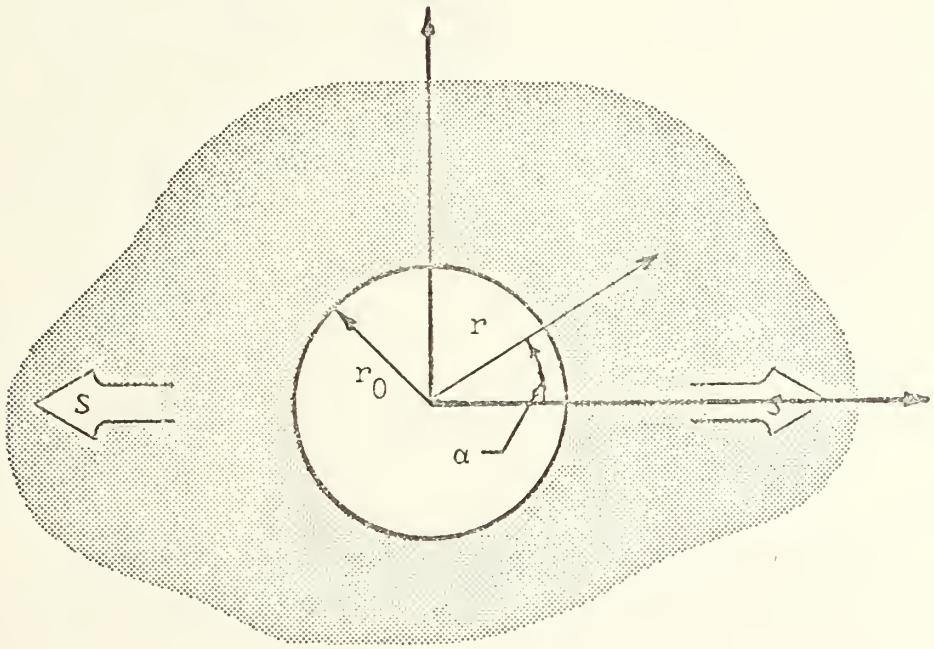


Fig. C2. Circular hole in an infinite plate

If the plate is large in comparison with the size of a circular hole the following relations hold:

$$\sigma_r = S[(1-R^2) + (1+3R^4-4R^2)\cos 2\alpha]/2 \qquad (C21)$$

$$\sigma_\alpha = S[(1+R^2) - (1+3R^4)\cos 2\alpha]/2 \qquad (C22)$$

$$\theta_0 = \sigma_r + \sigma_\alpha = S(1 - 2R^2 \cos 2\alpha) \qquad (C23)$$

where $R = r_0/r$ (C24)

Introducing the three-dimensional behavior, we have

$$(\sigma_\alpha) = \frac{\partial^2 \phi}{\partial r^2} = \frac{\partial^2 \phi_0}{\partial r^2} + cz^2 \frac{\partial^2 \theta_0}{\partial r^2} \qquad (C25)$$

and evaluating at the edge ($r = r_0$), we get

$$(\sigma_\alpha)_{edge} = \sigma_0 + \Delta\sigma_0$$

$$= [S-2S \cos 2\alpha] + [6S(z/r_0)^2 \nu \cos 2\alpha/(1+\nu)] \qquad (C26)$$

Table C1 shows evaluations of this formula at various points around the hole, in terms of the ratio λ/r_0 where λ is the half thickness.

TABLE C1

EDGE STRESS AROUND HOLE

α	σ_0/S	$\frac{\Delta\sigma_0}{S} \Big/ \frac{\lambda^2}{r_0^2}$	$\frac{\Delta\sigma_0}{\sigma_0} \Big/ \frac{\lambda^2}{r_0^2}$
0	-1	1.38	-1.38
15	-0.732	1.20	-1.64
30	0	0.69	∞
45	1	0	0
60	2	-0.69	-0.35
75	2.732	-1.20	-0.44
90	3	-1.38	-0.46

Inserting the numerical data as previously used: $r_0 = 0.2$, $\lambda = 0.125$; and solving at $\alpha = 90°$:

$$(\sigma_\alpha)_{edge} = 0.82\ \sigma_0 \qquad (C27)$$

Proceeding as before:

$$\overline{\theta}_0'' = 99.79$$

$$(\sigma_\alpha)_{ave} = 0.94\ (\sigma_\alpha)_{max} \qquad (C28)$$

A more precise analysis using Howland's [8] formulation yields essentially the same result. It is apparent that if a circular hole is used to infer the needed correction, as suggested in Section 3 above, SCF's reported for the U-shaped notch will be slightly high. This is apparently the procedure used by Flynn and Roll in their experiments.

4. NUMERICAL DIFFERENTIATION FROM 2D FEM

For this evaluation, we deal directly with the U-shaped notch. The same numerical data was used to facilitate direct comparisons. Our objective is to take the FEM computed values of $\sigma_x + \sigma_y$ at the nodes along the line normal to the notch, fit a polynomial through the values obtained and compute the second derivative in the normal direction. Figure C3 shows the notch.

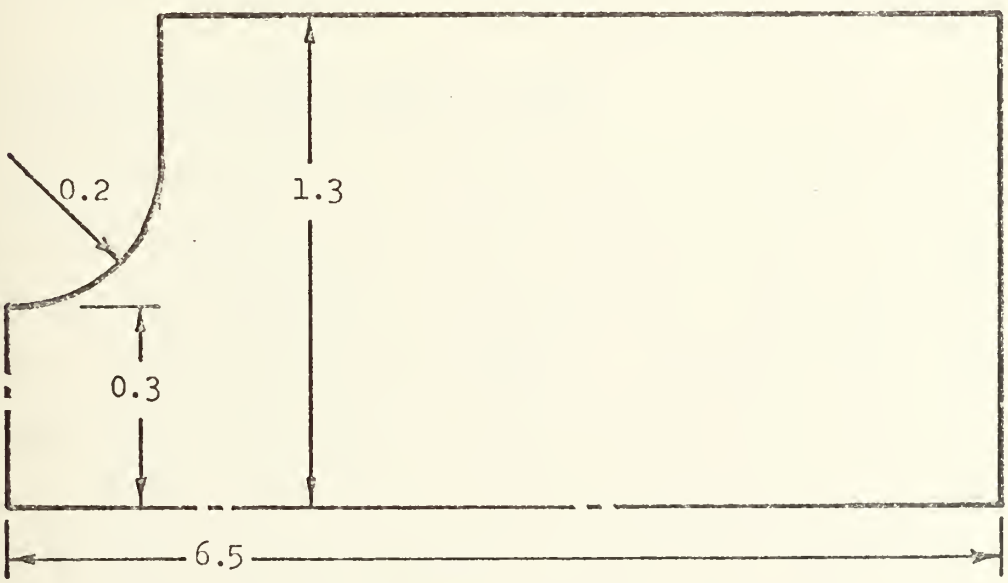


Fig. C3. Symmetric U-shaped notches (not to scale)

Examination of the stress-sum profile reveals that a smooth curve is not obtained even with the finest mesh used. (This is primarily because of a minor systematic inherent discrepancy between the FEM evaluation of stress at a midside node and at a corner node.) Choosing the element corner nodes only, a fourth degree, least square polynomial can be calculated. Examination of the second derivative of this polynomial reveals that it is slowly converging to some value with each successive refinement of the mesh. Viewing each of these results as a term in a convergent sequence, a convergence-accelerating procedure due to Shanks [14] was used to estimate θ_0'' at the base of the notch. The numerical result was:

$$\overline{\theta}_0'' \underset{\sim}{\sim} 10$$

$$(\sigma_x)_{ave} = 0.994\ (\sigma_x)_{max} \qquad \text{(C29)}$$

5. DIRECT COMPARISON FROM 3D FEM

The FEM program used in this study was TRISOP [10]. Quadratic elements were used for the solution. The mesh was generated by TRIMEG [1], modified in the same way as the two dimension mesh generator (PLIMEG), to progressively vary the element size. The mesh in the x-y plane was the same as that used in the 2D solutions. Two equal size elements were used for the half thickness in the z direction. Because of symmetry about the midthickness plane, it was sufficient to consider only half the plate. The half thickness of the bar (previously designated λ) was 0.125. Figure C4 shows the notch.

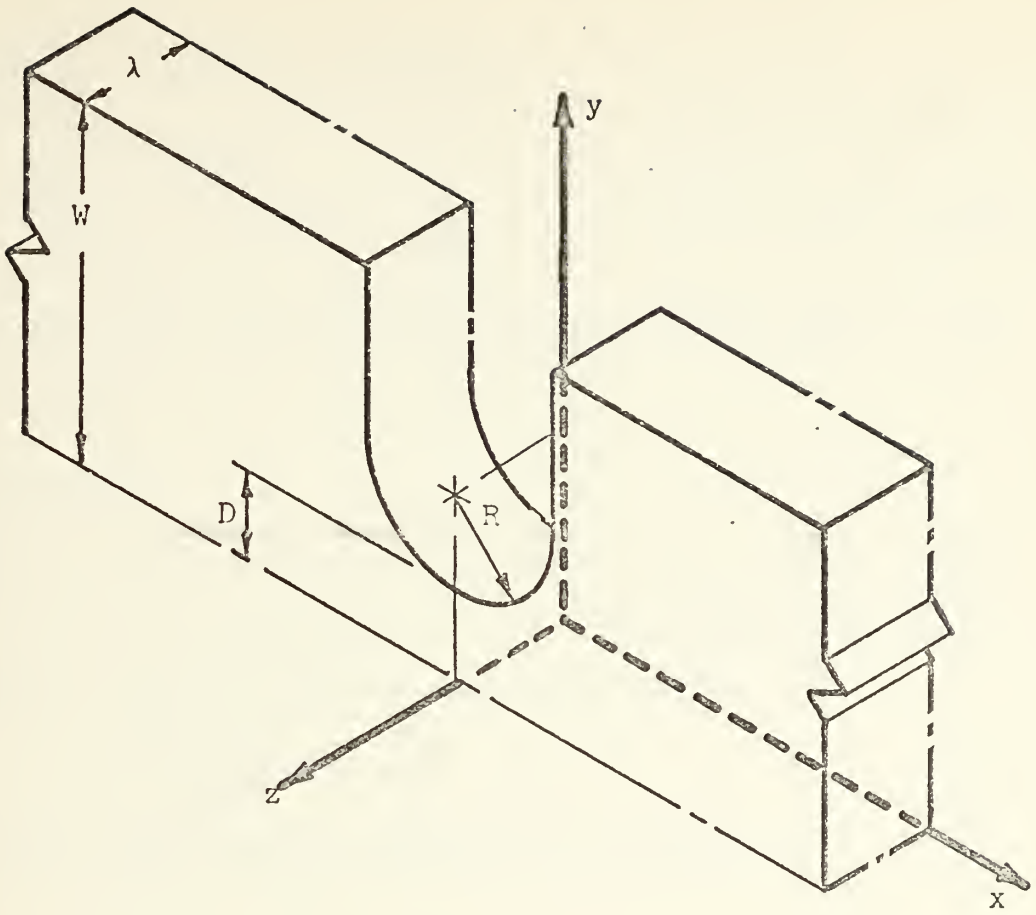


Fig. C4. U-shaped notches. (xy, yz, zx planes are planes of symmetry. The planes $z = \pm\lambda$ are outside faces, the planes $y = \pm w$ are the unnotched boundaries)

Each computational run involved nearly three and one-half hours of computer time. Because of this only a few runs were made to insure proper program operation and gather this data. Since a large data gathering task was not contemplated here, the mesh generator and FEM program were not coupled together. All data input to the mesh generator was manually prepared. All calculations subsequent to determination of nodal stresses was done outside the computer.

A parabola was fitted through the values σ_x at the corner nodes along the line $y = a = 0.3$. The result of this procedure,

cast again in familiar terms was

$$\bar{\theta}_0'' = 14.98$$

$$(\sigma_x)_{ave} = 0.991\ (\sigma_x)_{max} \qquad (C30)$$

The former value was inferred from the coefficients in the parabolic fit.

6. CONCLUSION

We are now in a position to compare the various results and estimate the importance of the three-dimensional effect. The aspect ratio of the bar considered here is

$$AR = \frac{\text{thickness}}{\text{width}} = \frac{0.25}{2.6} \simeq 0.1 \qquad (C31)$$

The results obtained are summarized in Table C2.

TABLE C2
Results of Three-Dimensional Analyses

Method	$\sigma_{ave}/\sigma_{max}$	$\bar{\theta}_0''$	Remarks
Neuber notch	0.982	18.85	Two derivatives of analytical stress-sum.
Hole in plate	0.94	99.79	Two derivatives of analytical stress-sum.
2D FEM	0.994	10	Two derivatives of the curve fit through stress-sum from numerical data.
3D FEM	0.991	14.98	Parabola fit through numerical computations of σ_x.

As can be seen from the results presented here, the effect of three-dimensionality is indeed small. The two-dimensional and three-dimensional FEM studies compare closely enough to permit extracting the third dimension effect from PLISOP. The extraction is not easy. However, the procedure outlined is readily accomplished with the computer and requires much less computational time than does the execution of TRISOP.

We see also from these results that an indiscriminate adjustment of photoelastically obtained values of SCF based on the SCF for a known geometry may cause overcorrection. The overcorrection is small, however, and the error causes conservative estimates, that is, one overestimates the ratio $(\sigma_{max}/\sigma_{ave})_{notch}$.

APPENDIX D

SUGGESTIONS FOR PRODUCING ADDITIONAL NUMERICAL RESULTS

Stress concentration factors for the four problems presented in this thesis can easily be calculated by using the program in Appendix F. For all four problems the useful range of the parameter D/W is from zero to one and the range of θ is from 0° to 90°. Note that either D/W = 1 or θ = 90° describes a bar with no notch and results in SCF = 1. D/W may be incremented in steps of 0.2 (program statements AA 01340 and AA 01200) and θ in steps of 15° (program statements AA 01380 and AA01180) for all problems to provide sufficient data. The incrementing step selected in the program for R/W is 0.05 (program statements AA 01360 and AA 01190).

The SCF's for problem 1 over the full range of D/W, all θ up to 60° and R/W from 0.1 to 0.5 are presented in Appendix A. To duplicate this information for the other three problems, all that is needed is three additional computer runs. The sample input deck of figure 13 of the text may be duplicated and used with a single change for each new problem. The entry on card 4 in columns 11-15 will be changed to 2, 3 and 4 for each successive run.

Each computer run will require 12.5 hours and will provide the raw data (a deck of punched cards, each listing the problem parameters and corresponding SCF) paralleling

the data plotted in Appendix A. A simple plotting routine will be needed to plot the data.

In Section III C of the text, we pointed out that the SCF at two points are of interest for problem 4, namely the point at the bottom of the notch and also the opposite point on the straight boundary. As presented, this program calculates only the first of these for each new problem. To make the information more complete, an investigator should add the card "Y(15) = SSJNT(19,1)" following PP 0930. Following SS 00090 add the card "SCF2 = SCF * Y(15)/Y(14)"; change present card SS 00130 by adding ",SCF2"; change present card SS 00140 to "FORMAT (6D15.8)".

APPENDIX E

PROGRAM USER'S MANUAL

This user's manual is intended to be used with the program listed in Appendix F.

A user familiar with neither computer coding or the Finite Element Method (FEM) of numerical solution can, with the aid of this manual:

1. execute the program listed to solve the four specific problems presented in the body of this thesis.

A user familiar with both computer coding and FEM, in addition to the above, can:

2. change the boundary conditions applied to the structure.
3. change the shape of the notch in a minor way.
4. change the program dimensions to suit the user's needs.
5. modify the program to solve a different, parameterized problem.

1. TO EXECUTE THE PROGRAM

The job control language cards needed are listed in Figure E1. No explanation of their make-up will be attempted in this manual other than to comment that some of the cards listed beginning with //FTXXF001 provide the direct access storage needed by the program. Others provide linkages needed to print, plot, and punch cards.

A sample input deck is included in Figure E1. The entries on the cards are listed in Table E1 in detail. Figure E2 (identical with Figure 9 in the text) is a sketch of the structure considered.

```
// (STANDARD JOB CARD)
// EXEC FORTCLG,REGION=690K
//FORT.SYSIN DD *


C         INSERT FORTRAN DECK HERE


//GO.FT06F001 DD SPACE=(CYL,(4,1))
//GO.FT07F001 DD SYSOUT=B,SPACE=(CYL,(5,1))
//GO.SYSPLOT  DD SYSOUT=C,UNIT=(SYSOUT,SEP=(FT05F001,FT06F001))
//GO.SYSPLOTS DD UNIT=SYSDA,SPACE=(TRK,(2,6))
//GO.FT10F001 DD UNIT=SYSDA,SPACE=(080,(0500,1)),DISP=(NEW,DELETE)
//GO.FT11F001 DD UNIT=SYSDA,SPACE=(080,(0001,1)),DISP=(NEW,DELETE)
//GO.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(10,1)),DISP=(NEW,DELETE)
//GO.SYSIN DD *
    00.3D0     0.50D0     30.0D0
     THIS IS A SAMPLE INPUT DECK -- CARD COLUMNS ARE INDICATED ON THE NEXT CARD
12345678901234567890123456789012345678901234567890123456789012345678901234567890
    4     8     1     2
    1     8    12     1    14    17     4     1    12              1.0       -1.0
    2     1    12    14    27    30    17     1     4              1.0
    5     8     3     4    17    20     7     1    12                        -1.0
    6     1     3    17    30    33    20     1    12
                1                    0     2
```

FIG. E1 JCL AND SAMPLE INPUT DECK

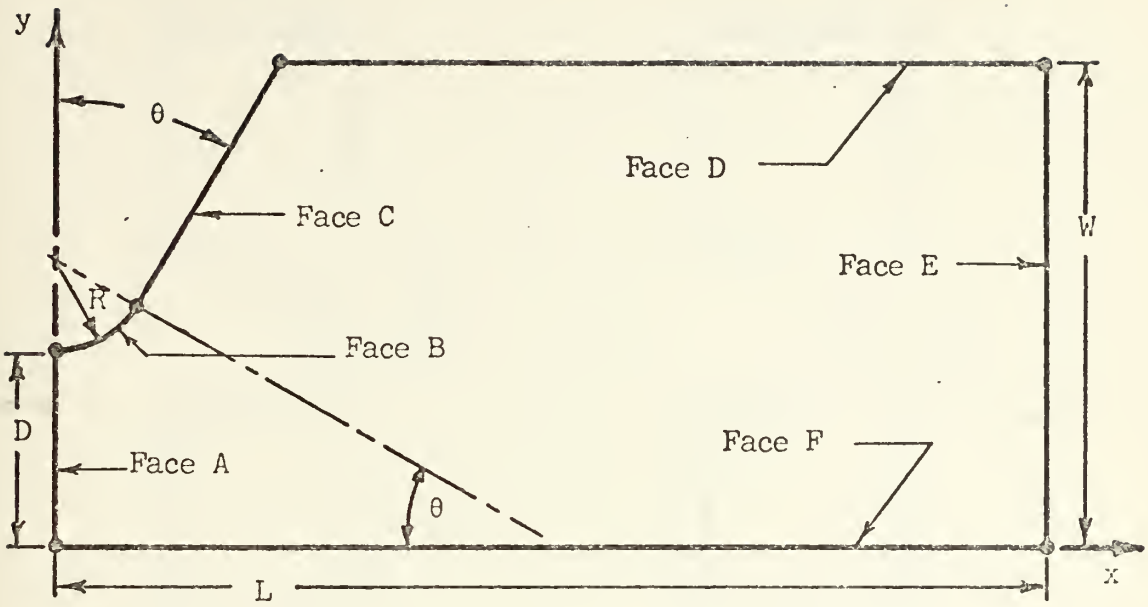


Fig. E2. Structure Faces

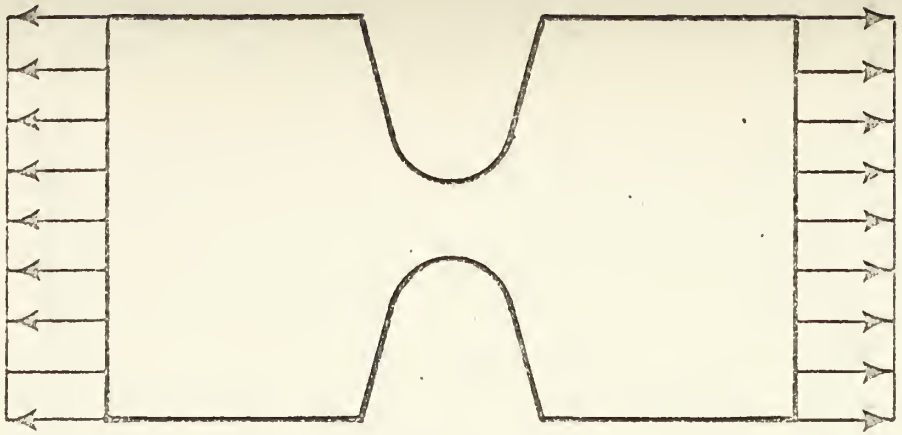
Figure E3 (identical with Figure 8 in the text) shows the four problems solved. The user must make some selections and can change other numbers; however some entries on these cards should not be changed. They are indicated by:

User must choose	-( )-
User may choose	( )
User may not choose	*( )*

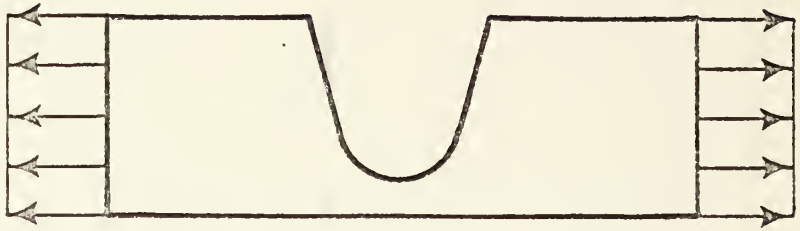
Some additional explanatory material may be found in the thesis by Adamek [1].

Notes to Accompany Figure E1 and Table E1.

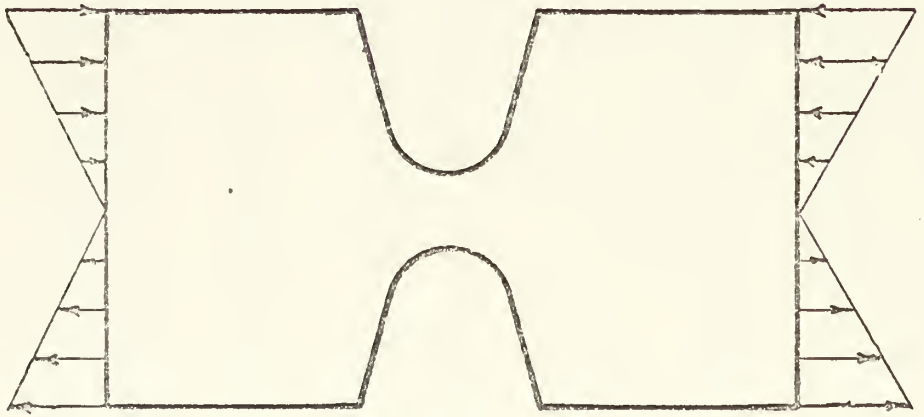
a. Card 4: If NPLOT= 1 or 2, only a single problem is solved. This is done to avoid unnecessarily drawing the same mesh



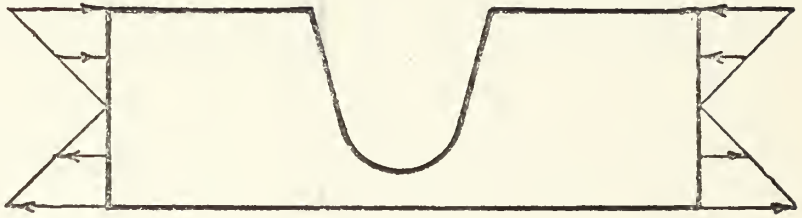
Problem 1



Problem 3



Problem 2



Problem 4

Fig. E3. The Four Problems

with only minor modification 225 times. This also provides the capability to stack as many problems as desired in one data run. If NPLOT $\neq$ 0, at the end of calculations for each problem, control returns to the reading section of MAIN to begin a new problem.

b. Program PLIMEG has an ordering of SEL's with which the present use conforms. This means that here we use only SEL's numbered 1, 2, 5, and 6. Cards 5, 6, 7, and 8 relate to SEL's numbered 1, 2, 5, and 6, in that order.

c. Cards 5-8: Element size increases with coordinate direction for positive EXPX and EXPY. For the smallest element to lie at the base of the notch EXPX should be positive and EXPY should be negative. If the values shown are not used the stresses calculated may be farther from the converged result than 1% shown in the body of this thesis.

d. Cards 5-8: ROWS must be the same for SEL's 1 and 5 and for SEL's 2 and 6. Likewise for EXPY.

e. Cards 5-8. CLMNS must be the same for SEL's 1 and 2 and SEL's 5 and 6. Likewise for EXPX.

f. Cards 5-8: Dimension statements in the program limit the ROWS and CLMNS as follows; ROWS: SEL1 + SEL2 $\leq$ 9; CLMNS: SEL1 + SEL5 $\leq$ 15.

g. Card 9: Card columns not listed are not used and should be left blank.

h. Card 9: For a discussion of NGP see reference [16], Chapter 8. For this formulation NGP = 2 is adequate.

TABLE E1

LIST AND DESCRIPTION OF INPUT DECK

Card Number	Card Columns	Variable Name	Explanation
1			problem parameters
	1-10	(RW)	radius/gross width ratio
	11-20	(DW)	net/gross width ratio
	21-30	(TH)	Notch semi-angle (degrees)
2			identification
	1-48	-(TITLE)-	as desired by user
3			additional identification
	1-48	-(TITLE)-	as desired by user
4			mesh parameters
	1-5	*(NSEL)*	number of SEL's
	6-10	*(NPT)*	number nodes per element
	11-15	-(NPUNCH)-	1 indicator for problem 1
			2 indicator for problem 2
			3 indicator for problem 3
			4 indicator for problem 4
	16-20	-(NPLOT)-	0 indicator for no plots
			1 indicator for printer plot
			2 indicator for CALCOM PLOT
5-8			super element deck
	1-5	*(SELNR)*	SEL identification number
	6-10	(ROWS)	number of columns (divisions in y direction) in this SEL
	11-15	(CLMNS)	number of columns (divisions in x direction) in this SEL

TABLE E1 (Continued)

Card Number	Card Columns	Variable Name	Explanation
	16-20	*(Node A)*	upper left corner node number
	21-25	*(Node B)*	lower left corner node number
	26-30	*(Node C)*	lower right corner node number
	31-35	*(Node D)*	upper right corner node number
	36-40	*(TYPE)*	material identification number
	41-45	*(NPTSB)*	number of boundary nodes in this SEL
	46-50	*(CON)*	code for interconnection between SEL – for use see thesis by Adamek [H]
	51-60	(EXPX)	exponential coefficient XI direction
	61-70	(EXPY)	exponential coefficient ETA direction
9			FEM parameters
	11-15	*(NMAT)*	number of materials in the structure
	26-30	(NSTRES)	2 dimension assumption made: 0 for plane stress 1 for plane strain
	31-35	(NGP)	number of Gauss points used in the integration

Computer time required may be estimated by knowing that the compile and link steps require approximately two minutes total and each problem executes in three minutes. Core requirement with this configuration is 690K. The core requirement may be reduced to 615K if cards with serial numbers GG 00530, GG 00740 and GG 00750 are removed from the deck. This removes the plotting capabilities from the program. If NPLOT= 1 or 2 is used, the program terminates by "end of file on unit 5" (the card reader is empty) after the last problem is solved.

2. TO CHANGE BOUNDARY CONDITIONS

Computations for the load boundary conditions and codes indicating the displacement specifications are entirely contained in subroutine COORD. The serial numbers on all FORTRAN statements used to calculate boundary conditions begin with the three digits EEX or EEY. These instructions assume that the user can calculate externally applied forces F_x and F_y to describe the loading desired.

Nodal displacement specifications may be imposed in any of several ways. Each specification is indicated by a code assigned to one specific node. The array NBC (454,2) is used to transmit the coded information. NBC (I,1) is the node number at which the I^{th} displacement specification applies and NBC (I,2) is the code for that specification taken from the following list.

1: A roller on an x axis (i.e., v = 0)

2: A roller on a y axis (i.e., u = 0)

3: A fixed node (i.e., u = v = 0)

A cumulative tally of the number of nodes where a displacement specification has been imposed is contained in the variable NPBC and the tally appears in the output.

At least one nodal displacement must be specified in each of the two coordinate directions in order to prevent rigid body motion which might result in meaningless outputs.

The cards which must be modified are serialized EEY. The actual changes to be made are left to the user.

3. TO CHANGE THE SHAPE OF THE NOTCH

Only a minor change can be accomplished easily. The notch shape is modified but no modification of the SEL arrangement is contemplated. That is, we only wish to relocate SEL nodes 1 through 7 (Fig. E4, identical with Fig. 12 in the text) and subsequently to generate an appropriate FEM mesh. It is assumed that the same problem parameters (R/W, D/W, θ) are used.

In subroutine PROBLM, the x and y coordinates of nodes 1 through 7 must be re-coded. The FORTRAN statements that must be changed can easily be identified by the user. Some computational short cuts may be useful under certain conditions. Before introducing the shortcuts, however, the user must understand the data string output by this subroutine.

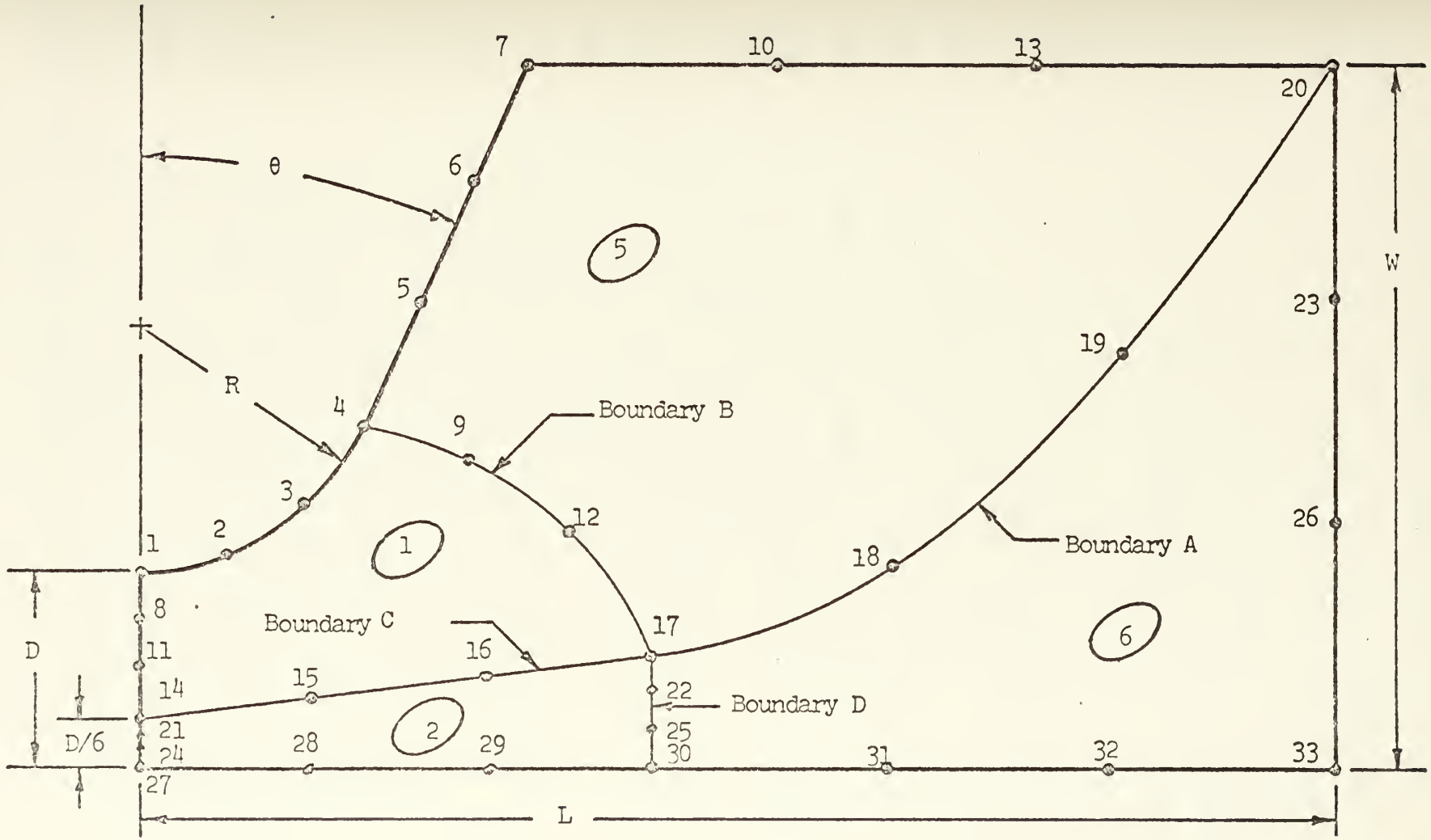


Fig. E4. Structure SEL's and boundary nodes. SEL identification numbers appear in ovals. (Not to scale)

We refer to each data string written on direct access device 10 as a node card which has the format (3I5, 2D25.16). The first three numbers (integers) are the number of the SEL (variable J), the node number (variable K) and the short-cutting code (Variable I or L). The other two numbers (Real*8) are the x and y coordinates of node K (variables X(K) and Y(K) respectively). The node cards are grouped by SEL number and the groups arranged in increasing numerical order. Within each SEL group the cards are arranged according to the SEL connectivity. That is, the node in the upper right hand corner first, followed by one card for each node used to describe the SEL, ordered in counter-clockwise fashion. In this program SEL's 1, 5, and 6 have 12 nodes each; SEL 2 has 4 nodes.

In what follows, some familiarity with PLIMEG and PLISOP is assumed. The coded numbers are described and/or calculated as shown in Table D2.

The shortcutting code referred to above may be used if the SEL boundary between any two corner nodes is a straight line. L=5 for 12 node elements (or L=4 for 8 node elements; not used in this program) causes PLIMEG (the mesh generator itself) to calculate the coordinates of these intermediate nodes. Therefore the coordinates need not be calculated in subroutine PROBLM. Card serial BB 00790 is an example of the use of the shortcutting code. Note that where the short-cutting code L is not used, I= 0 must be specified as has been done in card serial BB 00830.

The structure boundary nodes along the circular arc (points 1 through 4) are corrected (see page 38 of text) in subroutine COORD. A user modifying subroutine PROBLM must also recode the corrections made by cards serialized EEZ. The variables CORD(N, 1) and CORD(N, 2) are the x and y coordinates of node N respectively.

4. TO CHANGE DIMENSIONING

The cards listed in Table E2 include all of the controllable dimension statements in the program. Each is an exact reproduction of its parent card in the program except that the dimension numbers, where appropriate, have been replaced by a code: NR2 through NR15. By calculating the numbers represented by these codes, preparaing a new dimension deck and replacing the parent cards in the program with the new cards (on the basis of serial number) a user may tailor the program size to meet his needs.

The designations included in the name column are variable names used in PLIMEG and PLISOP. If several different versions of the dimension deck might be required, the computer terminal can be advantageously used. This can be done by reading the coded deck into the system, then using up to fourteen change commands to change the codes to the previously calculated numbers, and then ordering a new punched deck.

If several problems requiring different dimensions are to be solved, they can all be solved using a program with the largest of the dimensions. From the standpoint of

efficient computer usage, however, it is better to dimension the program to fit each job.

A rough estimate of core storage required (in K BYTES) may be arrived at by making the following computation:

$$\text{CORE} = \frac{2 * \text{NO4} * \text{NO10}}{128} + 180$$

5. TO MAKE MAJOR MODIFICATION

A major modification to the program involves, primarily, a complete rewrite of subroutine PROBLM. The guidance of Section 3, together with a good working knowledge of PLIMEG, can be extended to accomplish this. The user may be guided by carefully studying the preceding portions of this appendix.

TABLE E1

CONTROLLABLE DIMENSION STATEMENTS

```
C
C
C
```

```
      COMMON/MESH2/ NCR(NO6),NRR(NO7)                                    GGW00090
      COMMON/MESH3/ NCON(NO8,14)                                         GGW00100
      COMMCN/CORD1/CORD(NO4,4),ELCON(NO3,4),NBC(NO4,2)                   GGW00110
      CCMMON/SOL/BGK(NO9,NO10),ALOAD(NC9),ABGN                           HHW00820
      COMMCN/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(NO8,14),NSTRES,NGP,      HHW00830
     1LM(NO11),LJT(NO11),NTLD,LTBY,NTIN                                  HHW00840
      CCMMON/CORD1/COARD(NO4,2),CLOAD(NO4,2),ELCON(NO3,4),NBC(NO4,2)     HHW00850
      DIMENSION REACT(NO12,2),NCODE(NC4)                                 HHW00860
      EQUIVALENCE (BGK(1,1),REACT(1,1))                                  HHW00870
      MEL=NO8                                                            HHW00880
      MJT=NO4                                                            HHW00890
      MMT=NO3                                                            HHW00900
      MBD=NO10                                                           HHW00910
      CCMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(NO8,14),NSTRES,NGP,      IIW00050
     1LM(NO11),LJT(NO11),NTLD,LTBY,NTIN                                  IIW00060
      CCMMON/CORD1/COARD(NO4,2),CLOAD(NO4,2),ELCON(NO3,4),NBC(NO4,2)     IIW00070
      DIMENSION              STK(NO13,NO13),AK(NO13,NO13),B(3,NO13)      JJW00030
      CCMMCN/SOL/BGK(NO9,NO10),ALOAD(NC9),ABGN                           JJW00040
      CCMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(NO8,14),NSTRES,NGP,      JJW00050
     1LM(NO11),LJT(NO11),NTLD,LTBY,NTIN                                  JJW00060
      CCMMON/CORD1/COARD(NO4,2),CLOAD(NO4,2),ELCON(NO3,4),NBC(NO4,2)     JJW00070
      CCMMON CCORD(12,2),ELAST(3,3),SS(NO14,NO13),SN(NO14,NO13)          JJW00080
      DIMENSION AK(NO13,NO13),STK(NO13,NO13),B(3,NO13)                   LLW00470
      DIMENSION XI(NO15),AI(NO15),AIA(NO15,NO15)                         LLW00480
      CCMMON CCORD(12,2),ELAST(3,3),SS(NO14,NO13),SN(NO14,NO13)          LLW00490
      DIMENSION AJ(2,2),AJIN(2,2),DNX(2,NO11),W1(2,12),B(3,NO13)         MMW00060
     1,B1(NO13,3),AK(NO13,NO13)                                          MMW00070
      COMMON CCORD(12,2),ELAST(3,3),SS(NO14,NO13),SN(NO14,NO13)          MMW00080
      COMMCN/SOL/A(NO9,NO10),B(NO9),ABGN                                 NNW00170
      CCMMON/SOL/A(NO9,NO10),B(NO9),ABGN                                 OOW00150
      COMMCN/SOL/BGK(NO9,NO10), DISP(NC9),ABGN                           PPW00040
      CCMMCN/STAT/WR,DR,TH,W,X(33),Y(33),EXPX(NO2),EXPY(NO2)             PPW00050
      CCMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(NO8,14),NSTRES,NGP,      PPW00060
     1LM(NO11),LJT(NO11),NTLD,LTBY,NTIN                                  PPW00070
      CCMMCN/CORD1/COARD(NO4,2),CLOAD(NO4,2),ELCON(NO3,4),NBC(NO4,2)     PPW00080
      COMMCN CCORD(12,2),ELAST(3,3),SS(NO14,NO13),SN(NO14,NO13)          PPW00090
      DIMENSION SSJNT(NO4,7),SNJNT(NO4,7),SSEL(NO14),SNEL(NO14),         PPW00100
     1DSPEL(NO14)                                                        PPW00110
      COMMON/SOL/BGK(NO9,NO10),ALOAD(NC9),ABGN                           QQW00030
      COMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(NO8,14),NSTRES,NGP,      QQW00040
     1LM(NO11),LJT(NO11),NTLD,LTBY,NTIN                                  QQW00050
      COMMON/CORD1/COARD(NO4,2),CLOAD(NO4,2),ELCON(NO3,4),NBC(NO4,2)     QQW00060
      COMMON/SOL/BGK(NO9,NO10), DISP(NO9),ABGN                           RRW00050
      COMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(NO8,14),NSTRES,NGP,      RRW00060
     1LM(NO11),LJT(NO11),NTLD,LTBY,NTIN                                  RRW00070
      COMMCN/CORD1/COARD(NO4,2),CLOAD(NO4,2),ELCON(NO3,4),NBC(NO4,2)     RRW00080
      DIMENSION REACT(NO12,2)                                            RRW00090
      COMMON/STAT/WR,DR,TH,W,X(33),Y(33),EXPX(NO2),EXPY(NO2)             SSW00030
```

TABLE E1 (Continued)

EXPLANATIONS OF

DIMENSIONING CODES

Code	Name	Explanation
NO3	MAXMAT	The maximum number of materials to be considered
NO4	MAXNJT	The maximum number of nodes to be considered
NO6	NGCOL	The maximum number of columns in the basis SEL grid
NO7	NGROW	The maximum number of rows in the basic SEL grid
NO8	MAXNEL	The maximum number of elements to be considered
NO11	NJT	The maximum number of nodes per element (4, 8, or 12)
NO15	NGP	The maximum number of Gauss points to be used in the integrations (2 through 5)
NO2		NGROW * NGCOL * NO 11
NO5		(NGCOL + 1) * NGROW
NO9		2 * MAXNJT
NO10	NBAND	The half bandwidth of the stiffness matrix calculated from: IF NO11= 4, NBAND= 2 * (ROWS + 3) IF NO11= 8, NBAND= 2 * (3 * ROWS + 5) IF NO11= 12, NBAND= 2 * (5 * ROWS + 7) Note: ROWS is the maximum number of rows of elements in the completed mesh
NO12		MAXNJT + 1
NO13		2 * NO11
NO14		3 * NO11

```
C       PROGRAM NOTCH
C
C       PROGRAMMED, TESTED AND USED BY L. J. SMITH AT THE
C       NAVAL POSTGRADUATE SCHOOL, SPRING, 1974
C
C       THIS PROGRAM INCORPORATES THE PROGRAMS PLISOP, WRITTEN BY
C       G. CANTIN, MARCH,1972 AND PLIMEG, WRITTEN BY J. R. ADAMEK,
C       DEC., 1972.
C
C       RECORD OF SUBSEQUENT MODIFICATIONS SHOULD FOLLOW THIS LINE:
C
C
C
C       END OF RECORD OF MODIFICATIONS.
C
C       THIS PROGRAM CALCULATES THE STRESS CONCENTRATION FACTOR FOR
C       'V-NOTCHED' BARS (1 NOTCH AND 2 SYMMETRIC NOTCHES) BASED ON
C       GROSS WIDTH, NET WIDTH, BOTTOM RADIUS OF NOTCH AND ANGLE OF
C       THE 'V' SUBJECTED TO UNIFORM LOAD OR BENDING MOMENT.
C
C       A USER'S MANUAL APPEARS AS APPENDIX D OF THE THESIS 'STRESS
C       CONCENTRATION FACTORS FOR CIRCULAR NOTCHES' BY L. J. SMITH, JUNE 1974.
C
C     JCL CARDS REQUIRED:
C  *     (COLUMN ONE INDICATED BY ASTERISK)
C  //    (STANDARD JOB CARD)
C  // EXEC FORTCLG,REGION=690K
C  //FORT.SYSIN DD *
C
C       (INSERT THE FORTRAN DECK HERE)
C
C  //GO.FT06F001 DD SPACE=(CYL,(4,1))
C  //GO.FT07F001 DD SYSOUT=B,SPACE=(CYL,(5,1))
C  //GO.SYSPLOT DD SYSOUT=C,UNIT=(SYSOUT,SEP=(FT05F001,FT06F001))
C  //GO.SYSPLOTS DD UNIT=SYSDA,SPACE=(TRK,(2,6))
C  //GO.FT10F001 DD UNIT=SYSDA,SPACE=(080,(0500,1)),DISP=(NEW,DELETE)
C  //GO.FT11F001 DD UNIT=SYSDA,SPACE=(080,(0001,1)),DISP=(NEW,DELETE)
C  //GO.FT12F001 DD UNIT=SYSDA,SPACE=(CYL,(0010,1)),DISP=(NEW,DELETE)
C  //GO.SYSIN DD *
C
C       (INSERT THE INPUT DECK HERE)
C       (A SAMPLE INPUT DECK IS SHOWN BELOW)
C
```

```
C  *     (COLUMN ONE INDICATED BY ASTERISK)
C       0.1D0      0.5D0     30.0D0
C    THIS IS A SAMPLE INPUT DECK - 9X15 MESH
C    SYMMETRIC, U-SHAPED NOTCHES, UNIFORM LOAD
C       4     8     1     2
C       1     8    12     1    14    17     4     1    12              1.0       -1.0
C       2     1    12    14    27    30    17     1     4              1.0
C       5     8     3     4    17    20     7     1    12                        -1.0
C       6     1     3    17    30    33    20     1    12
C                   1                0     2
C
C         (END OF SAMPLE INPUT DECK.  PROBLEMS MAY BE STACKED BY USING)
C         (MORE THAN ONE SUCH INPUT DECK AT THIS PCINT.  EXECUTION)
C         (TERMINATES EITHER BY BOMBING OR NORMAL TERMINATION.  SEE USER'S)
C         (MANUAL, APPENDIX D OF THESIS.  THE LAST CARD, FOLLOWING)
C         (INPUT DECK OR DECKS, IS A 'SLASH-STAR'.)
C
C     EXPLANATION OF THE DATA DECK:
C     (NUMERALS IN PARENTHESES INDICATE VALUES EMPLOYED IN WORK DONE
C     IN THE AFOREMENTIONED THESIS.)
C                                                                          AA 00090
C     CARD 1 (3F10.2) PROBLEM PARAMETERS                                   AA 00100
C       1-10  RW:     RADIUS / UNDISTURBED WIDTH                  (0.1)    AA 00110
C      11-20  DW:     WAIST WIDTH / UNDISTURBED WIDTH             (0.5)    AA 00120
C      21-30  TH:     ANGLE OF INCLINATION (DEGREES)             (30.0)    AA 00130
C                                                                          AA 00140
C     CARDS 2 & 3 (6A8) EACH CARD  TITLE                                   AA 00150
C                                                                          AA 00160
C     CARD 4 (4I5) MESH PARAMETERS                                         AA 00170
C       1- 5  NSEL:   NUMBER OF SUPER ELEMENTS                    (4)     AA 00180
C       6-10  NPT:    NUMBER OF NODES PER ELEMENT                  (8)     AA 00190
C      11-15  NPUNCH:1 FOR SYMMETRIC - UNIFORM LOAD                        AA 00200
C                    2 FOR SYMMETRIC - BENDING MOMENT                      AA 00210
C                    3 FOR ONE NOTCH - UNIFORM LOAD                        AA 00220
C                    4 FOR ONE NOTCH - BENDING MOMENT                      AA 00230
C      16-20  NPLOT: 0 FOR NO PLOTS                                        AA 00240
C                    1 FOR PRINTER PLOT                                    AA 00250
C                    2 FOR CALCOM PLOT                                     AA 00260
C                                                                          AA 00270
```

```
C      CARD 5-(NSEL+4) (10I5,2F10.2) ONE PER SUPER ELEMENT                    AA 00280
C        1- 5   SELNR: SUPER ELEMENT IDENTIFICATION NUMBER                    AA 00290
C        6-10   ROWS:  NUMBER OF ROWS IN SUPER ELEMENT                        AA 00300
C       11-15   CLMNS: NUMBER OF COLUMNS IN SUPER ELEMENT                     AA 00310
C       16-20   NODE A:                                  A     D              AA 00320
C                         SUPER ELEMENT                 *****                AA 00330
C       21-25   NODE B:   CONNECTIVITY                  *   *                AA 00340
C                         NUMBERS                       *   *                AA 00350
C       26-30   NODE C:   COUNTERCLOCKWISE              *   *                AA 00360
C                         (SEE NOTE)                    *****                AA 00370
C       31-35   NODE D:                                 B     C              AA 00380
C                       NOTE: SEL NUMBERING IS ARBITRARY. HOWEVER,           AA 00390
C                       A NODE COMMON TO TWO OR MORE SELS MUST               AA 00400
C                       RETAIN ITS NUMBER.                                   AA 00410
C       36-40   TYPE:   MATERIAL ID NUMBER (ALWAYS 1)                (1)     AA 00420
C       41-45   NPTSB:  NUMBER OF BOUNDARY NODES FOR THIS SEL             (0) AA 00430
C                       (NPTSB = 4, 8 OR 12)                                 AA 00440
C       46-50   CON:    IF SEL NOT CONNECTED TO ADJACENT SEL                 AA 00450
C                       AT MID SIDE NODES, ENTER ADJACENT SEL                AA 00460
C                       NUMBER HERE. (NORMALLY BLANK)                        AA 00470
C       51-60   EXPX:   EXPONENTIAL COEFFF. - XI DIRECTION                   AA 00480
C       61-70   EXPY:   EXPONENTIAL COEFF. - ETA DIRECTION                   AA 00490
C                       NOTE: ELEMENT SIZE INCREASES WITH                    AA 00500
C                       CO-ORDINATE DIRECTION FOR POSITIVE                   AA 00510
C                       EXPX AND EXPY                                        AA 00520
C                                                                            AA 00530
C      CARD (NSEL + 5) (10I5) STRESS ANALYSIS PARAMETERS                     AA 00540
C        1-10           NOT USED (LEAVE BLANK)                               AA 00550
C       11-15   NMAT:   ALWAYS 1                                    (1)      AA 00560
C       26-30   NSTRES: 0 FOR PLANE STRESS                          (0)      AA 00570
C                       1 FOR PLANE STRAIN                                   AA 00580
C       31-35   NGP:    NUMBER OF GAUSS POINTS DESIRED             (2)      AA 00590
C                       IN THE INTEGRATION (2 THRU 5)                        AA 00600
C       36-50           NOT USED (LEAVE BLANK)                               AA 00610
C                                                                            AA 00620
       IMPLICIT REAL*8 (A-H,O-Z)                                             AA 00630
       COMMON/INT/NPT,NEL,NPUNCH,NSTOP,NJT,NPLOT,NKIND,NSEL,NGROW,NGCOL,     AA 00640
      1          NCLOAD,NPBC                                                 AA 00650
       COMMON/TITL/TITLE(12)                                                 AA 00660
C                                                                            AA 00670
C       FOR 9 X 15 QUADRATIC ELEMENT MESH - NOTCH PROBLEMS                   AA 00680
       COMMON/STAT/WR,DR,TH,W,X(33),Y(33),EXPX(48),EXPY(48)                  AAW00690
       COMMON/CORD1/CORD(454,4),ELCON(1,4),NBC(454,2)                        AAW00700
       COMMON/MESH1/ NSCON(20,9),MS(20,8),MEL(20,4)                          AAW00710
       COMMON COORD(12,2),ELAST(3,3),SS(24,16),SN(24,16)                     AAW00720
C                                                                            AA 00730
     7 REWIND 11                                                             AA 00740
       READ(5,15)RW,DW,TH                                                    AA 00750
```

```
      A=RW                                                                  AA 00760
      B=DW                                                                  AA 00770
   15 FORMAT(3F10.2)                                                        AA 00780
      READ (5,10) (TITLE(I),I=1,12)                                         AA 00790
      WRITE (6,20) (TITLE(I),I=1,6)                                         AA 00800
      WRITE (6,22) (TITLE(I),I=7,12)                                        AA 00810
20    FORMAT ('1',6A8)                                                      AA 00820
22    FORMAT (1X,6A8)                                                       AA 00830
      READ (5,11) NSEL,NPT,NPUNCH,NPLOT                                     AA 00840
      NKIND=NPT/4                                                           AA 00850
      DO 1000 N=1,NSEL                                                      AA 00860
      READ (5,11)   I,(NSCON(I,J),J=1,9),EXPX(I),EXPY(I)                    AA 00870
1000  WRITE(11,11)  I,(NSCON(I,J),J=1,9)                                    AA 00880
10    FORMAT (6A8)                                                          AA 00890
11    FORMAT (10I5,2F10.2)                                                  AA 00900
      W=1.3D0                                                               AA 00910
      IJK=0                                                                 AA 00920
      NR=NSCON(1,1)+NSCON(2,1)                                              AA 00930
      NC=NSCON(1,2)                                                         AA 00940
      GO TO (101,102,103),NKIND                                             AA 00950
 101  I1=0                                                                  AA 00960
      I2=NR+2                                                               AA 00970
      I3=(NC-1)* (I2-1)+1                                                   AA 00980
      I4=0                                                                  AA 00990
      I5=I2-1                                                               AA 01000
      I6=0                                                                  AA 01010
      GO TO 104                                                             AA 01020
 102  I1=(NR*2)+2                                                           AA 01030
      I2=(NR*3)+3                                                           AA 01040
      I3=(NC-1)*(I2-1)+1                                                    AA 01050
      I4=(NC-1)*(I2-1)+I1                                                   AA 01060
      I5=I2-1                                                               AA 01070
      I6=0                                                                  AA 01080
      GO TO 104                                                             AA 01090
 103  I1=(NR*3)+2                                                           AA 01100
      I2=(NR*5)+4                                                           AA 01110
      I3=(NC-1)*(I2-1)+1                                                    AA 01120
      I4=(NC-1)*(I2-1)+I2                                                   AA 01130
      I5=I2-1                                                               AA 01140
      I6=NR+1                                                               AA 01150
 104  WRITE(11,2999) I1,I2,I3,I4,I5,I6                                      AA 01160
2999  FORMAT(6I10)                                                          AA 01170
      DO 1 I=1,5                                                            AA 01180
      DO 2 L=1,10                                                           AA 01190
      DO 3 K=1,5                                                            AA 01200
      WR=1/RW                                                               AA 01210
      R=W*RW                                                                AA 01220
      DR=WR*DW                                                              AA 01230
```

```
      CALL PRCBLM                                                                  AA 01240
      IF (X(1).EQ.10.D0) GO TO 3                                                   AA 01250
      CALL PLIMEG                                                                  AA 01260
      IF(IJK.NE.0) GO TO 6                                                         AA 01270
      IJK=IJK+1                                                                    AA 01280
      READ(5,11   )NNN,NNN,NMAT,   NNN, NNN,NSTRES,NGP,NTLD,NTBY,NTIN             AA 01290
      WRITE(11,11)NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN              AA 01300
    6 CALL PLISOP                                                                  AA 01310
      CALL ANS                                                                     AA 01320
      IF(NPLOT.GT.0) GO TO 7                                                       AA 01330
    3 DW=DW+0.2D0                                                                  AA 01340
      DW=B                                                                         AA 01350
    2 RW=RW+0.05D0                                                                 AA 01360
      RW=A                                                                         AA 01370
    1 TH=TH+15.D0                                                                  AA 01380
 9000 STOP                                                                         AA 01390
      END                                                                          AA 01400
```

```
      SUBROUTINE PROBLM                                                            BB 00010
C                                                                                  BB 00020
      IMPLICIT REAL*8 (A-H,O-Z)                                                    BB 00030
      COMMON/STAT/WR,DR,TH,W,X(33),Y(33),EXPX(48),EXPY(48)                         BBW00040
C                                                                                  BB 00050
      REWIND 10                                                                    BB 00060
      PI=3.141592653590                                                            BB 00070
      R=W/WR                                                                       BB 00080
      D=DR*R                                                                       BB 00090
      WRITE(6,2000) WR,DR,TH,W,D,R                                                 BB 00100
      TH=(PI*TH)/540.D0                                                            BB 00110
      RD=R+D                                                                       BB 00120
      X(1)=0.D0                                                                    BB 00130
      Y(1)=D                                                                       BB 00140
      Y(7)=W                                                                       BB 00150
      X(14)=0.D0                                                                   BB 00160
      Y(14)=D/3.D0                                                                 BB 00170
      X(20)=5.D0*W                                                                 BB 00180
      Y(20)=W                                                                      BB 00190
      X(27)=0.D0                                                                   BB 00200
      Y(27)=0.D0                                                                   BB 00210
      X(33)=5.D0*W                                                                 BB 00220
      Y(33)=0.D0                                                                   BB 00230
      Y(30)=0.D0                                                                   BB 00240
      X(2)=R*DSIN(PI/6.D0-TH)                                                      BB 00250
      Y(2)=RD-R*DCOS(PI/6.D0-TH)                                                   BB 00260
```

```
      TH=2.D0*TH                                                        BB 00270
      X(3)=R*DSIN(2.D0*PI/6.D0-TH)                                      BB 00280
      Y(3)=RD-R*DCOS(2.D0*PI/6.D0-TH)                                   BB 00290
      TH=3.D0*TH/2.D0                                                   BB 00300
      X(4)=R*DCOS(TH)                                                   BB 00310
      Y(4)=RD-R*DSIN(TH)                                                BB 00320
      IF((Y(7)-Y(1)).LE.0.1D-3) GO TO 7                                 BB 00330
      IF((Y(7)-Y(4)).LE.0.1D-3) GO TO 4                                 BB 00340
      X(7)=X(4)+(W-Y(4))*DTAN(TH)                                       BB 00350
      X(5)=X(4)+(X(7)-X(4))/3.D0                                        BB 00360
      X(6)=X(4)+(X(7)-X(4))*2.D0/3.D0                                   BB 00370
      Y(5)=Y(4)+(Y(7)-Y(4))/3.D0                                        BB 00380
      Y(6)=Y(4)+(Y(7)-Y(4))*2.D0/3.D0                                   BB 00390
    6 TH=180.D0*TH/PI                                                   BB 00400
      IF(X(7).GE.X(20)) GO TO 3                                         BB 00410
      X(17)=W+R                                                         BB 00420
      X(30)=X(17)                                                       BB 00430
      DX=(X(20)-X(17))/3.D0                                             BB 00440
      X(18)=X(17)+DX                                                    BB 00450
      X(19)=X(18)+DX                                                    BB 00460
      C=(W-D/6.D0)/(25.D0*W**2)                                         BB 00470
      B=0.D0                                                            BB 00480
      DO 1 I=17,19                                                      BB 00490
    1 Y(I)=D/6.D0+B*X(I)+C*X(I)**2                                      BB 00500
      C=(Y(17)-Y(4))/(X(17)**2-X(4)**2-2.D0*(X(17)-X(4))*X(4))          BB 00510
      B=-2.D0*X(4)*C                                                    BB 00520
      A=Y(4)-X(4)*B-X(4)**2.D0*C                                        BB 00530
      DX=(X(17)-X(4))/3.D0                                              BB 00540
      X(9)=X(4)+DX                                                      BB 00550
      X(12)=X(9)+DX                                                     BB 00560
      DO 2 I=9,12,3                                                     BB 00570
    2 Y(I)=A+B*X(I)+C*X(I)**2                                           BB 00580
 2000 FORMAT(' ',20X,'W/R=       ',G10.2,/,                            BB 00590
     1       ' ',20X,'D/R=       ',G10.2,/,                            BB 00600
     2       ' ',20X,'THETA=     ',G10.2,/,                            BB 00610
     3       ' ',20X,'WIDTH=     ',G10.2,/,                            BB 00620
     4       ' ',20X,'DEPTH=     ',G10.2,/,                            BB 00630
     5       ' ',20X,'RADIUS=    ',G10.2)                              BB 00640
      L=5                                                               BB 00650
      A=0.D0                                                            BB 00660
      I=0                                                               BB 00670
      J=1                                                               BB 00680
      K=4                                                               BB 00690
 9998 FORMAT(3I5,2D25.16)                                               BB 00700
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00710
      K=3                                                               BB 00720
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00730
      K=2                                                               BB 00740
```

```
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00750
      K=1                                                               BB 00760
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00770
      K=8                                                               BB 00780
      WRITE(10,9998)J,K,L,A,A                                           BB 00790
      K=11                                                              BB 00800
      WRITE(10,9998)J,K,L,A,A                                           BB 00810
      K=14                                                              BB 00820
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00830
      K=15                                                              BB 00840
      WRITE(10,9998)J,K,L,A,A                                           BB 00850
      K=16                                                              BB 00860
      WRITE(10,9998)J,K,L,A,A                                           BB 00870
      K=17                                                              BB 00880
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00890
      K=12                                                              BB 00900
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00910
      K=9                                                               BB 00920
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00930
      J=2                                                               BB 00940
      K=17                                                              BB 00950
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00960
      K=14                                                              BB 00970
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 00980
      K=27                                                              BB 00990
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01000
      K=30                                                              BB 01010
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01020
      J=11                                                              BB 01030
      K=7                                                               BB 01040
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01050
      K=6                                                               BB 01060
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01070
      K=5                                                               BB 01080
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01090
      K=4                                                               BB 01100
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01110
      K=9                                                               BB 01120
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01130
      K=12                                                              BB 01140
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01150
      K=17                                                              BB 01160
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01170
      K=18                                                              BB 01180
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01190
      K=19                                                              BB 01200
      WRITE(10,9998)J,K,I,X(K),Y(K)                                     BB 01210
      K=20                                                              BB 01220
```

```
      WRITE(10,9998)J,K,I,X(K),Y(K)                                      BB 01230
      K=13                                                               BB 01240
      WRITE(10,9998)J,K,L,A,A                                            BB 01250
      K=10                                                               BB 01260
      WRITE(10,9998)J,K,L,A,A                                            BB 01270
      J=12                                                               BB 01280
      K=20                                                               BB 01290
      WRITE(10,9998)J,K,I,X(K),Y(K)                                      BB 01300
      K=19                                                               BB 01310
      WRITE(10,9998)J,K,I,X(K),Y(K)                                      BB 01320
      K=18                                                               BB 01330
      WRITE(10,9998)J,K,I,X(K),Y(K)                                      BB 01340
      K=17                                                               BB 01350
      WRITE(10,9998)J,K,I,X(K),Y(K)                                      BB 01360
      K=22                                                               BB 01370
      WRITE(10,9998)J,K,L,A,A                                            BB 01380
      K=25                                                               BB 01390
      WRITE(10,9998)J,K,L,A,A                                            BB 01400
      K=30                                                               BB 01410
      WRITE(10,9998)J,K,I,X(K),Y(K)                                      BB 01420
      K=31                                                               BB 01430
      WRITE(10,9998)J,K,L,A,A                                            BB 01440
      K=32                                                               BB 01450
      WRITE(10,9998)J,K,L,A,A                                            BB 01460
      K=33                                                               BB 01470
      WRITE(10,9998)J,K,I,X(K),Y(K)                                      BB 01480
      K=26                                                               BB 01490
      WRITE(10,9998)J,K,L,A,A                                            BB 01500
      K=23                                                               BB 01510
      WRITE(10,9998)J,K,L,A,A                                            BB 01520
      RETURN                                                             BB 01530
    3 WRITE(6,3000)                                                      BB 01540
      X(1)=10.D0                                                         BB 01550
 3000 FORMAT(' THETA TOO LARGE')                                         BB 01560
      RETURN                                                             BB 01570
    4 ALFA=DARCOS((RD-W)/R)                                              BB 01580
      IF(ALFA.LT.(PI/10.D0)) GO TO 7                                     BB 01590
      X(2)=R*DSIN(0.3D0*ALFA)                                            BB 01600
      X(3)=R*DSIN(0.6D0*ALFA)                                            BB 01610
      X(4)=R*DSIN(0.9D0*ALFA)                                            BB 01620
      X(5)=R*DSIN(0.9D0*ALFA+(ALFA/30.D0))                               BB 01630
      X(6)=R*DSIN(0.9D0*ALFA+(ALFA/15.D0))                               BB 01640
      X(7)=R*DSIN(ALFA)                                                  BB 01650
      DO 5 I=1,6                                                         BB 01660
    5 Y(I)=RD-DSQRT(R*R-X(I)*X(I))                                       BB 01670
      GO TO 6                                                            BB 01680
    7 WRITE(6,4000)                                                      BB 01690
      X(1)=10.D0                                                         BB 01700
```

```
      TH=180.D0*TH/PI                                                   BB 01710
 4000 FORMAT(' DEPTH TOO LARGE')                                        BB 01720
      RETURN                                                            BB 01730
      END                                                               BB 01740



      SUBROUTINE PLIMEG                                                 CC 00010
C                MESH GENERATING PROGRAM FOR 'PLISOP'                   CC 00020
C     CODED BY J.R.ADAMEK, DECEMBER 1972, NAVAL POSTGRADUATE SCHOOL      CC 00030
C     MODIFIED BY L.J.SMITH, MAY 1974, NAVAL POSTGRADUATE SCHOOL         CC 00040
C                                                                       CC 00050
C        THE NECESSARY INPUT FOR THIS PROGRAM IS AS FOLLOWS:            CC 00060
C            (PASSED VIA MAIN)                                          CC 00070
C                                                                       CC 00080
C  CARD 1 AND 2, TITLE, FORMAT (6A8) EACH CARD.                         CC 00090
C                                                                       CC 00100
C     COL. 1 TO 48   BRIEF TITLE OF PROBLEM. USER'S NAME AND/OR BOX NO. CC 00110
C                    MUST BE INCLUDED ON ONE OF THE CARDS IF CALCOMP     CC 00120
C                    OPTION INCLUDED. (SEE CARD 3)                      CC 00130
C                                                                       CC 00140
C                                                                       CC 00150
C  CARD 3, MESH PARAMENTERS, FORMAT (4I5).                              CC 00160
C                                                                       CC 00170
C     CCL. 1 TO  5   (NSEL):  NUMBER OF SUPER ELEMENTS.                 CC 00180
C                                                                       CC 00190
C          6 TO 10   (NPT):   THE NUMBER OF NODAL POINTS PER ELEMENT.   CC 00200
C                    NPT= 4,8 OR 12                                     CC 00210
C                                                                       CC 00220
C         11 TO 15   (NPUNCH): IF THIS IS ZERO OR BLANK, NO CARDS WILL  CC 00230
C                    BE PUNCHED. IF NPUNCH IS DIFFERENT FROM ZERO A DECKCC 00240
C                    FOR CONNECTIVITY AND X,Y COORDINATES OF ALL JOINTS CC 00250
C                    WILL BE PUNCHED. NPUNCH BLANK OR ZERO SHOULD ALWAYSCC 00260
C                    BE USED UNTIL ONE IS SATISFIED WITH THE MESH.      CC 00270
C                                                                       CC 00280
C         16 TO 20   (NPLOT):  IF THIS IS ZERO OR BLANK, NO PLOTS WILL  CC 00290
C                    BE MADE. IF NPLOT IS (1) A PLOT WILL BE OBTAINED ONCC 00300
C                    THE PRINTER(UTPLOT). IF NPLOT IS (2) A PLOT WILL BECC 00310
C                    OBTAINED ON CALCOMP PLOTTER.                       CC 00320
C                    NPLOT SHOULD BE (1) UNTIL SATISFIED WITH MESH.     CC 00330
C                    ALL PLOTS WILL BE PLOTTED IN FIRST QUADRANT ONLY.  CC 00340
C                                                                       CC 00350
C                                                                       CC 00360
C  SUPER ELEMENT DECK, TOTAL OF NSEL CARDS, FORMAT (10I5), LOAD IN      CC 00370
C                      ASCENDING ORDER OF SUPER ELEMENT NUMBERS.        CC 00380
C                                                                       CC 00390
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C     COL. 1 TO  5   (SEL NO.) SUPER ELEMENT IDENTIFICATION NUMBER.      CC 00400
C                                                                        CC 00410
C          6 TO 10   (ROW):   NUMBER OF ROWS IN SUPER ELEMENT.           CC 00420
C                                                                        CC 00430
C         11 TO 15   (COL):   NUMBER OF COLUMNS IN SUPER ELEMENT.        CC 00440
C                                                                        CC 00450
C         16 TO 20   (NODE  A):   ************************************CC 00460
C                                 *                          A     D   *CC 00470
C         21 TO 25   (NODE  B):   *    SUPER ELEMENT          *****    *CC 00480
C                                 *     CONNECTIVITY          *   *    *CC 00490
C         26 TO 30   (NODE  C):   *   COUNTERCLOCKWISE        *   *    *CC 00500
C                                 *      (SEE NOTE)           *****    *CC 00510
C         31 TO 35   (NODE  D):   *                          B     C   *CC 00520
C                                 ************************************CC 00530
C                                                                        CC 00540
C                    NOTE: NUMBERING OF SUPER-ELEMENT CORNER NODES IS    CC 00550
C                    ARBITRARY. HOWEVER, CORNER NODES WHICH ARE          CC 00560
C                    CONNECTED TO OTHER CORNER NODES, MUST HAVE          CC 00570
C                    THE SAME NUMBER.                                    CC 00580
C                                                                        CC 00590
C         36 TO 40   (TYPE):  MATERIAL IDENTFICATION NUMBER FOR          CC 00600
C                             SUPER ELEMENT.(SEE 'PLISOP' INST)          CC 00610
C                                                                        CC 00620
C         41 TO 45   (NPTSB): NUMBER OF SUPER ELEMENT BOUNDARY NODES.    CC 00630
C                             NPTSB = 4,8 OR 12                          CC 00640
C                                                                        CC 00650
C         46 TO 50   (CON):   IF SUPER ELEMENT IS CONNECTED TO AN        CC 00660
C                             ADJACENT SUPER ELEMENT BELOW OR TO THE     CC 00670
C                             RIGHT ONLY AT THE CORNER NODES AND NOT IN  CC 00680
C                             BETWEEN, ENTER ADJACENT SUPER ELEMENT      CC 00690
C                             NUMBER HERE. OTHERWISE LEAVE BLANK.        CC 00700
C                                                                        CC 00710
C                                                                        CC 00720
C  SUPER ELEMENT BOUNDARY DECK, TOTAL OF NPTB CARDS,   (3I5,2G25.17)     CC 00730
C                                                                        CC 00740
C                    NPTB=SUM OF BOUNDARY NODES OF ALL SUPER ELEMENTS.   CC 00750
C                    ORDER IS COUNTER CLOCK-WISE STARTING AT NODE COMMON CC 00760
C                    WITH (XI,ETA)=(1,1). IF MORE THAN ONE SUPER ELEMENT,CC 00770
C                    LOAD ITS BOUNDARY AFTER THAT OF FIRST SUPER ELEMENT CC 00780
C                    IN SAME NODAL ORDER AS ABOVE. ORDER OF SUPER ELEMENTS CC 00790
C                    SAME AS FOR SUPER ELEMENT DECK.                     CC 00800
C                                                                        CC 00810
C     COL. 1 TO  5   (SEL NO.):  SUPER ELEMENT IDENTIFICATION NUMBER.    CC 00820
C                                                                        CC 00830
C          6 TO 10   (NODE NO.): ARBITRARY NUMBERING OF SUPER ELEMENT    CC 00840
C                                BOUNDARY NODES. MAY OR MAY NOT BE THE   CC 00850
C                                SAME NUMBERING SCHEME AS                CC 00860
C                                FOR CONNECTIVITY.                       CC 00870
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C                                                                       CC 00880
C          11 TO 15   (NCT):    TYPE OF COORDINATES. IF THIS FIELD IS   CC 00890
C                     BLANK OR ZERO, CARTESIAN COORDINATES ARE USED.    CC 00900
C                     IF NCT IS (1), THE X COORDINATE IS INTERPRETED    CC 00910
C                     AS THE RADIUS VECTOR AND THE Y COORDINATE AS THE  CC 00920
C                     POLAR ANGLE IN DEGREES WITH CENTER OF CURVATURE   CC 00930
C                     AT THE ORIGIN. NCT = (2) OR (3) IS INTERPRETED THECC 00940
C                     SAME AS FOR NCT=(1), EXCEPT CENTER OF CURVATURE   CC 00950
C                     NOT AT THE ORIGIN. IF NCT IS (2), A NEW DATA CARD CC 00960
C                     IS INSERTED AFTER PRESENT ONE. ONLY REQUIRED FOR  CC 00970
C                     FIRST NODE WITH THIS PARTICULAR CENTER OF         CC 00980
C                     CURVATURE. ** FORMAT (2G25.17) **                 CC 00990
C                                                                       CC 01000
C                     COL. 1 TO 25  X COORD. OF CENTER OF CURVATURE     CC 01010
C                                                                       CC 01020
C                          26 TO 50  Y COORD. OF CENTER OF CURVATURE.    CC 01030
C                                                                       CC 01040
C                     NCT IS (3) FOR CENTER OF CURVATURE NOT AT ORIGIN  CC 01050
C                     FOR REMAINING NODES WITH THIS PARTICULAR CENTER   CC 01060
C                     OF CURVATURE. NCT IS (4) FOR MID SIDE NODE OF EIGHTCC 01070
C                     NODAL BOUNDARY WITH PARTICULAR SIDE LINEAR. NCT IS CC 01080
C                     (5) FOR MID SIDE NODE OF TWELVE NODAL BOUNDARY    CC 01090
C                     WITH PARTICULAR SIDE LINEAR.. LOAD ZERO OR BLANK  CC 01100
C                     FOR X AND Y COORDINATES WHEN NCT IS (4) OR (5).   CC 01110
C                                                                       CC 01120
C          16 TO 40   X COORDINATE OF NODE, REAL*8                      CC 01130
C                                                                       CC 01140
C          41 TO 65   Y COORDINATE OF NODE, REAL*8                      CC 01150
C                                                                       CC 01160
      IMPLICIT REAL*8 (A-H,O-Z)                                         CC 01170
      DIMENSION NCT(48)                                                 CCW01180
      COMMON/TITL/TITLE(12)                                             CCW01190
      COMMON/INT/NPT,NEL,NPUNCH,NSTOP,NJT,NPLOT,NKIND,NSEL,NGROW,NGCOL, CCW01200
     1           NCLOAD,NPBC                                            CCW01210
      COMMON/MESH1/ NSCON(20,9),MS(20,8),MEL(20,4)                      CCW01220
      COMMON/CORD1/CORD(454,4),ELCON(1,4),NBC(454,2)                    CCW01230
      COMMON/CORD2/BOUND(48,2)                                          CCW01240
      COMMON/STAT/WR,DR,TH,W,X(33),Y(33),EXPX(48),EXPY(48)              CCW01250
      DATA MAXBJT/454/                                                  CCW01260
      NGROW=4                                                           CCW01270
      NGCOL=4                                                           CCW01280
      REWIND 10                                                         CC 01290
      REWIND 11                                                         CC 01300
C     WRITE (6,20) (TITLE(I),I=1,6)                                     CC 01310
C     WRITE (6,22) (TITLE(I),I=7,12)                                    CC 01320
C     WRITE (6,35)                                                      CC 01330
C     WRITE (6,33) NSEL,NPT,NPUNCH,NPLOT                                CC 01340
 10   FORMAT (6A8)                                                      CC 01350
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 11    FORMAT (10I5)                                                          CC 01360
 20    FORMAT ('1',6A8)                                                       CC 01370
 22    FORMAT (1X,6A8)                                                        CC 01380
 25    FORMAT (3I5,2D25.16)                                                   CC 01390
 26    FORMAT (2G25.17)                                                       CC 01400
 28    FORMAT (4X,I3,6X,I3,5X,I3,2(3X,G25.15))                                CC 01410
 29    FORMAT (/,44X,'CENTER OF CURVATURE',/,27X,G25.15,3X,G25.15,/)          CC 01420
 30    FORMAT (//,' BOUNDARY DATA',//,'   SEL NO.',2X,'NODE NO.',2X,          CC 01430
      1'NCT',9X,'X COORDINATE',16X,'Y COORDINATE',//)                         CC 01440
 31    FORMAT (10X,I3,8X,8(I3,3X),I3)                                         CC 01450
 33    FORMAT (10X,I3,3X,I3,4X,I3,5X,I3)                                      CC 01460
 34    FORMAT (///,' SUPER ELEMENT DATA',//,9X,'SEL NO.',6X,'ROW',            CC 01470
      13X,'COL',3X,'(A)',3X,'(B)',3X,'(C)',3X,'(D)',2X,'TYPE',2X,'NPTSB',CC 01480
      22X,'CON',/)                                                           CC 01490
 35    FORMAT (///,' MESH PARAMETERS',//,10X,'NSEL',3X,'NPT',2X,'NPUNCH',CC 01500
      13X,'NPLCT',/)                                                         CC 01510
       NSTOP=0                                                               CC 01520
       NKIND=NPT/4                                                           CC 01530
       K=1                                                                   CC 01540
       I1=NGCOL+1                                                            CC 01550
       DO 40 I=1,I1                                                          CC 01560
       DO 39 J=1,NGROW                                                       CC 01570
       DO 36 L=1,9                                                           CC 01580
 36    NSCON(K,L)=0                                                          CC 01590
       DO 37 M=1,8                                                           CC 01600
 37    MS(K,M)=0                                                             CC 01610
       DO 38 N=1,2                                                           CC 01620
 38    MEL(K,N)=0                                                            CC 01630
 39    K=K+1                                                                 CC 01640
 40    CONTINUE                                                              CC 01650
C                                                                            CC 01660
C         SUPER ELEMENT DATA                                                 CC 01670
C                                                                            CC 01680
       NPTSBT=0                                                              CC 01690
C      WRITE (6,34)                                                          CC 01700
       DO 45 N=1,NSEL                                                        CC 01710
       IF (NPTSBT.GT.MAXBJT) NSTOP=2                                         CC 01720
       READ(11,11)   I,(NSCON(I,J),J=1,9)                                    CC 01730
       IF (N.EQ.NSEL) NLAST=I                                                CC 01740
C      WRITE (6,31)  I,(NSCON(I,J),J=1,9)                                    CC 01750
 45    NPTSBT=NPTSBT+NSCON(I,8)                                              CC 01760
C                                                                            CC 01770
C         BOUNDARY DATA                                                      CC 01780
C                                                                            CC 01790
C      WRITE (6,30)                                                          CC 01800
       NSELT=NGCOL*NGROW                                                     CC 01810
       I3=1                                                                  CC 01820
       NBS1=0                                                                CC 01830
```

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      DO 70 K=1,NSELT                                                    CC 01840
      IF (NSCON(K,2).EQ.0) GO TO 70                                      CC 01850
      I4=NSCON(K,8)+I3-1                                                 CC 01860
      DO 60 I=I3,I4                                                      CC 01870
      READ (10,25)   NBSEL,NBNODE,NCT(I),(BOUND(I,J),J=1,2)              CC 01880
C     WRITE (6,28)  NBSEL,NBNODE,NCT(I),(BOUND(I,J),J=1,2)               CC 01890
      IF (NBSEL.LT.NBS1) NSTOP=1                                         CC 01900
      NBS1=NBSEL                                                         CC 01910
      NCTI=NCT(I)                                                        CC 01920
      GO TO (46,46,46),NCTI                                              CC 01930
      GO TO 47                                                           CC 01940
 46   PHI=.31415926535897901*BOUND(I,2)/1.8D2                            CC 01950
      RAD=BOUND(I,1)                                                     CC 01960
      BOUND(I,1)=RAD*DCOS(PHI)                                           CC 01970
      BOUND(I,2)=RAD*DSIN(PHI)                                           CC 01980
 47   GO TO (50,48,49),NCTI                                              CC 01990
      GO TO 50                                                           CC 02000
 48   READ (5,26) XCC,YCC                                                CC 02010
C     WRITE (6,29) XCC,YCC                                               CC 02020
 49   BOUND(I,1)=BOUND(I,1)+XCC                                          CC 02030
      BOUND(I,2)=BOUND(I,2)+YCC                                          CC 02040
 50   II=I-1                                                             CC 02050
      III=I-2                                                            CC 02060
      IIII=I3-1                                                          CC 02070
      IF (II.LE.0) II=1                                                  CC 02080
      IF (III.LE.0) III=1                                                CC 02090
      IF ((K.EQ.NLAST).AND.(I.EQ.I4)) GO TO 51                           CC 02100
      GO TO 52                                                           CC 02110
 51   II=I                                                               CC 02120
      III=I-1                                                            CC 02130
      IIII=I                                                             CC 02140
 52   IF(NCT(II).EQ.4) GO TO 53                                          CC 02150
      IF((NCT(III).EQ.5).AND.(NCT(II).EQ.5)) GO TO 55                    CC 02160
      GO TO 60                                                           CC 02170
 53   NB1=II-1                                                           CC 02180
      NB2=II+1                                                           CC 02190
      IF (II.EQ.IIII) NB2=II-7                                           CC 02200
      BOUND(II,1)=(BOUND(NB1,1)+BOUND(NB2,1))/2.0D0                      CC 02210
      BOUND(II,2)=(BOUND(NB1,2)+BOUND(NB2,2))/2.0D0                      CC 02220
      GO TO 60                                                           CC 02230
 55   NB1=II-2                                                           CC 02240
      NB2=II-1                                                           CC 02250
      NB3=II+1                                                           CC 02260
      IF (II.EQ.IIII) NB3=II-11                                          CC 02270
      BMIDX=BOUND(NB1,1)-BOUND(NB3,1)                                    CC 02280
      BMIDY=BOUND(NB1,2)-BOUND(NB3,2)                                    CC 02290
      BOUND(III,1)=BOUND(NB1,1)-BMIDX/3.0D0                              CC 02300
      BOUND(III,2)=BOUND(NB1,2)-BMIDY/3.0D0                              CC 02310
```

```
      BOUND(II,1)=BOUND(NB1,1)-2*BMIDX/3.0D0                            CC 02320
      BOUND(II,2)=BOUND(NB1,2)-2*BMIDY/3.0D0                            CC 02330
 60   CONTINUE                                                          CC 02340
      I3=I4+1                                                           CC 02350
 70   CONTINUE                                                          CC 02360
      IF (NSTOP.EQ.1) GO TO 75                                          CC 02370
      IF (NSTOP.EQ.2) GO TO 90                                          CC 02380
      GO TO 85                                                          CC 02390
 75   WRITE (6,80)                                                      CC 02400
C                                                                       CC 02410
 80   FORMAT (///,'  DATA REJECTED***SUPER ELEMENT NUMBERS NOT IN ASCENDCC 02420
     1ING ORDER FOR BOUNDARY INPUT DATA.',/,                            CC 02430
     2'**************************************************',///)        CC 02440
      RETURN                                                            CC 02450
 90   WRITE (6,95) MAXBJT                                               CC 02460
C                                                                       CC 02470
 95   FORMAT (///,'  DATA REJECTED***NUMBER OF BOUNDARY NODES GREATER THCC 02480
     1AN',I5,/,'**************************************************',/)CC 02490
      RETURN                                                            CC 02500
 85   NSTOP=0                                                           CC 02510
      CALL CONN                                                         CC 02520
      IF (NSTOP.EQ.1) RETURN                                            CC 02530
      CALL COORD                                                        CC 02540
      IF (NPLOT.NE.0) CALL GRID                                         CC 02550
 9000 RETURN                                                            CC 02560
      END                                                               CC 02570
```



```
      SUBROUTINE CONN                                                   DD 00010
C        THIS SUBROUTINE DETERMINES ELEMENT CONNECTIVITY FOR 4,8 AND 12 DD 00020
C        NODAL ELEMENTS. OUTPUT IS PRINTED AND AN OPTION TO PUNCH IN    DD 00030
C        FORMAT COMPATIBLE WITH "PLISOP" IS INCORPORATED.               DD 00040
C                                                                       DD 00050
      IMPLICIT REAL*8 (A-H,O-Z)                                         DD 00060
      INTEGER*4 INF1,INF2,INF3,INFB                                     DD 00070
      DIMENSION NLC(14,3), LCON(14)                                     DD 00080
      COMMON/INT/NPT,NEL,NPUNCH,NSTOP,NJT,NPLOT,NKIND,NSEL,NGROW,NGCOL, DD 00090
     1           NCLOAD,NPBC                                            DD 00100
      COMMON/MESH1/ NSCON(20,9),MS(20,8),MEL(20,4)                      DDW00110
      COMMON/MESH2/ NCR(4),NRR(4)                                       DDW00120
      COMMON/MESH3/ NCON(135,14)                                        DDW00130
      DATA  MAXNEL/135/,MAXNJT/454/                                     DDW00140
      DATA NLC/4,1,2,3,5,6,7,8,9,10,11,12,13,14,7,8,1,2,3,4,5,6,9,10,   DD 00150
     111,12,13,14,10,11,12,1,2,3,4,5,6,7,8,9,13,14/                    DD 00160
 20   FORMAT (///,' CONNECTIVITY MATRIX',//,' ELEMENT NUMBER ',64X,' KINDD 00170
```

```
     1D',1X,'TYPE',//)                                                          DD 00180
  21 FORMAT (//,' NUMBER OF ELEMENTS=',I4,//,' NUMBER OF JOINTS=',I4)           DD 00190
  25 FORMAT (5X,I3,11X,14I5)                                                    DD 00200
  26 FORMAT (15I4)                                                              DD 00210
C                                                                               DD 00220
C        NUMBER OF ROWS AND COLUMNS IN EACH GRID COLUMN.                        DD 00230
C                                                                               DD 00240
      K=1                                                                       DD 00250
      REWIND 10                                                                 DD 00260
      DO 60 I=1,NGCOL                                                           DD 00270
      NRR(I)=0                                                                  DD 00280
      NCR(I)=0                                                                  DD 00290
      DO 50 J=1,NGROW                                                           DD 00300
      NRR(I)=NRR(I)+NSCON(K,1)                                                  DD 00310
      IF (NCR(I).NE.0) GO TO 50                                                 DD 00320
      IF (NSCON(K,2).NE.0) NCR(I)=NSCON(K,2)                                    DD 00330
  50  K=K+1                                                                     DD 00340
  60  CONTINUE                                                                  DD 00350
C                                                                               DD 00360
C        ZERO CONNECTIVITY MATRIX                                               DD 00370
C                                                                               DD 00380
      DO 80 I=1,MAXNEL                                                          DD 00390
      DO 70 J=1,14                                                              DD 00400
  70  NCON(I,J)=0                                                               DD 00410
  80     CONTINUE                                                               DD 00420
C                                                                               DD 00430
C        SUPER ELEMENT CONNECTIVITY                                             DD 00440
C                                                                               DD 00450
      K=1                                                                       DD 00460
      DO 100 I=1,NGCOL                                                          DD 00470
      IF(NCR(I).EQ.0) GO TO 95                                                  DD 00480
      J1=NGROW                                                                  DD 00490
      IF (I.EQ.NGCOL) J1=NGROW-1                                                DD 00500
      DO 90 J=1,J1                                                              DD 00510
      IF (NSCON(K,1).EQ.0) GO TO 90                                             DD 00520
      L=K+1                                                                     DD 00530
      M=K+NGROW                                                                 DD 00540
      IF (NSCON(K,4).NE.NSCON(L,3)) GO TO 86                                    DD 00550
      MS(K,4)=1                                                                 DD 00560
      MS(L,1)=1                                                                 DD 00570
  86  IF (NSCON(K,5).NE.NSCON(L,6)) GO TO 87                                    DD 00580
      MS(K,5)=1                                                                 DD 00590
      MS(L,8)=1                                                                 DD 00600
  87  IF (I.EQ.NGCOL) GO TO 90                                                  DD 00610
      IF (NSCON(K,6).NE.NSCON(M,3)) GO TO 88                                    DD 00620
      MS(K,7)=1                                                                 DD 00630
      MS(M,2)=1                                                                 DD 00640
  88  IF (NSCON(K,5).NE.NSCON(M,4)) GO TO 90                                    DD 00650
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      MS(K,6)=1                                                          DD 00660
      MS(M,3)=1                                                          DD 00670
 90   K=K+1                                                              DD 00680
      GO TO 100                                                          DD 00690
 95   K=K+NGROW                                                          DD 00700
 100  CONTINUE                                                           DD 00710
C                                                                        DD 00720
C        ELEMENT CONNECTIVITY                                            DD 00730
C                                                                        DD 00740
      NC1=1                                                              DD 00750
      KK=1                                                               DD 00760
      MM1=1                                                              DD 00770
      II=0                                                               DD 00780
      DO 500 I=1,NGCOL                                                   DD 00790
      L1=NCR(I)                                                          DD 00800
      IF (L1.EQ.0) GO TO 470                                             DD 00810
      II=II+1                                                            DD 00820
      DO 450 L=1,L1                                                      DD 00830
      LL1=NKIND                                                          DD 00840
      LL2=NKIND+1                                                        DD 00850
      IF (L.EQ.L1) LL1=LL2                                               DD 00860
      DO 430 LL=1,LL1                                                    DD 00870
      L3=0                                                               DD 00880
      IF ((LL.EQ.1).AND.(L.EQ.1)) L3=1                                   DD 00890
      NKIND1=NKIND                                                       DD 00900
      IF ((LL.NE.1).AND.(LL.NE.LL2)) NKIND1=1                            DD 00910
      K=KK                                                               DD 00920
      M1=MM1                                                             DD 00930
      DO 400 J=1,NGROW                                                   DD 00940
      IF (NSCON(K,1).EQ.0) GO TO 390                                     DD 00950
      KA=K-1                                                             DD 00960
      KB=NSCON(K,9)-1                                                    DD 00970
      KL=K-NGROW                                                         DD 00980
      KL1=KL-1                                                           DD 00990
      KL2=KL+1                                                           DD 01000
      KR=K+NGROW                                                         DD 01010
      KR1=KR-1                                                           DD 01020
      IF (LL.NE.LL2) GO TO 145                                           DD 01030
      IF (((MS(K,7).EQ.0).AND.(MS(K,8).EQ.1)).AND.(MS(KA,6).EQ.1))       DD 01040
     1  NC1=NC1-1                                                        DD 01050
      IF (((MS(K,6).EQ.1).AND.(MS(K,7).EQ.1))                           DD 01060
     1.AND.(KR.NE.NSCON(K,9))) GO TO 135                                 DD 01070
      GO TO 140                                                          DD 01080
 135  M1=M1+NSCON(K,1)                                                   DD 01090
      GO TO 390                                                          DD 01100
 140  IF (MS(K,7).EQ.1) NC1=NC1-1                                        DD 01110
 145  M2=M1+NSCON(K,1)-1                                                 DD 01120
      IF (M2.GT.MAXNEL) GO TO 700                                        DD 01130
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      DO 350 M=M1,M2                                                    DD 01140
      IF (LL.NE.1) GO TO 148                                            DD 01150
      IF (L.NE.1) GO TO 147                                             DD 01160
      IF (M.EQ.M1) MEL(K,1)=M                                           DD 01170
      IF (M.EQ.M2) MEL(K,2)=M                                           DD 01180
  147 IF (L.NE.L1) GO TO 148                                            DD 01190
      IF (M.EQ.M1) MEL(K,3)=M                                           DD 01200
      IF (M.EQ.M2) MEL(K,4)=M                                           DD 01210
  148 GO TO (150,200,250),NKIND                                         DD 01220
C                                                                       DD 01230
C        LINEAR ELEMENTS                                                DD 01240
C                                                                       DD 01250
  150 GO TO (160,185),LL                                                DD 01260
  160 NCON(M,1)=NC1                                                     DD 01270
      NCON(M,2)=NC1+1                                                   DD 01280
      IF (L.EQ.1) GO TO 165                                             DD 01290
      NCON(MM2,4)=NCON(M,1)                                             DD 01300
      NCON(MM2,3)=NCON(M,2)                                             DD 01310
      MM2=MM2+1                                                         DD 01320
      GO TO 300                                                         DD 01330
  165 IF(II.EQ.1) GO TO 300                                             DD 01340
      IF ((MS(K,2).EQ.0).AND.(MS(K,3).EQ.0)) GO TO 300                  DD 01350
      IF (M.NE.M1) GO TO 171                                            DD 01360
      MM3=MEL(KL,3)                                                     DD 01370
      MM4=MEL(K,1)                                                      DD 01380
  171 IF ((MS(K,2).EQ.1).AND.(MS(K,3).EQ.1)) GO TO 172                  DD 01390
      GO TO 300                                                         DD 01400
  172 IF (K.EQ.NSCON(KL,9)) GO TO 300                                   DD 01410
      NCON(MM3,4)=NCON(MM4,1)                                           DD 01420
      NCON(MM3,3)=NCON(MM4,2)                                           DD 01430
      MM3=MM3+1                                                         DD 01440
      MM4=MM4+1                                                         DD 01450
      GO TO 300                                                         DD 01460
  185 NCON(M,4)=NC1                                                     DD 01470
      NCON(M,3)=NC1+1                                                   DD 01480
      GO TO 300                                                         DD 01490
C                                                                       DD 01500
C        QUADRATIC ELEMENTS                                             DD 01510
C                                                                       DD 01520
  200 GO TO (205,225,245),LL                                            DD 01530
  205 NCON(M,1)=NC1                                                     DD 01540
      NCON(M,2)=NC1+1                                                   DD 01550
      NCON(M,3)=NC1+2                                                   DD 01560
      IF (L.EQ.1) GO TO 210                                             DD 01570
      NCON(MM2,7)=NCON(M,1)                                             DD 01580
      NCON(MM2,6)=NCON(M,2)                                             DD 01590
      NCON(MM2,5)=NCON(M,3)                                             DD 01600
      MM2=MM2+1                                                         DD 01610
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      GO TO 300                                                          DD 01620
  210 IF(II.EQ.1) GO TO 300                                              DD 01630
      IF ((MS(K,2).EQ.0).AND.(MS(K,3).EQ.0)) GO TO 300                   DD 01640
      IF (M.NE.M1) GO TO 215                                             DD 01650
      MM3=MEL(KL,3)                                                      DD 01660
      MM4=MEL(K,1)                                                       DD 01670
  215 IF ((MS(K,2).EQ.1).AND.(MS(K,3).EQ.1)) GO TO 220                   DD 01680
      GO TO 300                                                          DD 01690
  220 IF (K.EQ.NSCON(KL,9)) GO TO 300                                    DD 01700
      NCON(MM3,7)=NCON(MM4,1)                                            DD 01710
      NCON(MM3,6)=NCON(MM4,2)                                            DD 01720
      NCON(MM3,5)=NCON(MM4,3)                                            DD 01730
      MM3=MM3+1                                                          DD 01740
      MM4=MM4+1                                                          DD 01750
      GO TO 300                                                          DD 01760
  225 NCON(M,8)=NC1                                                      DD 01770
      NCON(M,4)=NC1+1                                                    DD 01780
      GO TO 300                                                          DD 01790
  245 NCON(M,7)=NC1                                                      DD 01800
      NCON(M,6)=NC1+1                                                    DD 01810
      NCON(M,5)=NC1+2                                                    DD 01820
      GO TO 300                                                          DD 01830
C                                                                        DD 01840
C        CUBIC ELEMENTS                                                  DD 01850
C                                                                        DD 01860
  250 GO TO (255,285,290,295),LL                                         DD 01870
  255 NCON(M,1)=NC1                                                      DD 01880
      NCON(M,2)=NC1+1                                                    DD 01890
      NCON(M,3)=NC1+2                                                    DD 01900
      NCON(M,4)=NC1+3                                                    DD 01910
      IF (L.EQ.1) GO TO 260                                              DD 01920
      NCON(MM2,10)=NCON(M,1)                                             DD 01930
      NCON(MM2, 9)=NCON(M,2)                                             DD 01940
      NCON(MM2, 8)=NCON(M,3)                                             DD 01950
      NCON(MM2, 7)=NCON(M,4)                                             DD 01960
      MM2=MM2+1                                                          DD 01970
      GO TO 300                                                          DD 01980
  260 IF(II.EQ.1) GO TO 300                                              DD 01990
      IF ((MS(K,2).EQ.0).AND.(MS(K,3).EQ.0)) GO TO 300                   DD 02000
      IF (M.NE.M1) GO TO 265                                             DD 02010
      MM3=MEL(KL,3)                                                      DD 02020
      MM4=MEL(K,1)                                                       DD 02030
  265 IF ((MS(K,2).EQ.1).AND.(MS(K,3).EQ.1)) GO TO 270                   DD 02040
      GO TO 300                                                          DD 02050
  270 IF (K.EQ.NSCON(KL,9)) GO TO 300                                    DD 02060
      NCON(MM3,10)=NCON(MM4,1)                                           DD 02070
      NCON(MM3, 9)=NCON(MM4,2)                                           DD 02080
      NCON(MM3, 8)=NCON(MM4,3)                                           DD 02090
```

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      NCON(MM3, 7)=NCON(MM4,4)                                                  DD 02100
      MM3=MM3+1                                                                 DD 02110
      MM4=MM4+1                                                                 DD 02120
      GO TO 300                                                                 DD 02130
285   NCON(M,12)=NC1                                                            DD 02140
      NCON(M, 5)=NC1+1                                                          DD 02150
      GO TO 300                                                                 DD 02160
290   NCON(M,11)=NC1                                                            DD 02170
      NCON(M, 6)=NC1+1                                                          DD 02180
      GO TO 300                                                                 DD 02190
295   NCON(M,10)=NC1                                                            DD 02200
      NCON(M, 9)=NC1+1                                                          DD 02210
      NCON(M, 8)=NC1+2                                                          DD 02220
      NCON(M, 7)=NC1+3                                                          DD 02230
300   NC1=NC1+NKIND1                                                            DD 02240
      NCON(M,13)=NKIND                                                          DD 02250
      NCON(M,14)=NSCON(K,7)                                                     DD 02260
350   CONTINUE                                                                  DD 02270
      IF ((II.NE.1).AND.(L3.EQ.1)) GO TO 355                                    DD 02280
      GO TO 375                                                                 DD 02290
355   IF ((MS(K,2).EQ.0).AND.(MS(K,3).EQ.0)) GO TO 375                          DD 02300
      MM5=MEL(K,2)                                                              DD 02310
      MM6=MEL(KL,4)                                                             DD 02320
      MM7=MEL(KL1,4)                                                            DD 02330
      MM8=MEL(KL2,3)                                                            DD 02340
      MM9=MEL(KL,3)                                                             DD 02350
      MM10=MEL(K,1)                                                             DD 02360
      GO TO (360,365,370),NKIND                                                 DD 02370
360   IF (MS(K,2).NE.1) GO TO 362                                               DD 02380
      NCON(MM9,4)=NCON(MM10,1)                                                  DD 02390
      IF (MS(KL,8).EQ.1) NCON(MM7,3)=NCON(MM9,4)                                DD 02400
362   IF (MS(K,3).NE.1) GO TO 375                                               DD 02410
      NCON(MM6,3)=NCON(MM5,2)                                                   DD 02420
      IF (MS(KL,5).EQ.1) NCON(MM8,4)=NCON(MM6,3)                                DD 02430
      GO TO 375                                                                 DD 02440
365   IF (MS(K,2).NE.1) GO TO 367                                               DD 02450
      NCON(MM9,7)=NCON(MM10,1)                                                  DD 02460
      IF (MS(KL,8).EQ.1) NCON(MM7,5)=NCON(MM9,7)                                DD 02470
367   IF (MS(K,3).NE.1) GO TO 375                                               DD 02480
      NCON(MM6,5)=NCON(MM5,3)                                                   DD 02490
      IF (MS(KL,5).EQ.1) NCON(MM8,7)=NCON(MM6,5)                                DD 02500
      GO TO 375                                                                 DD 02510
370   IF (MS(K,2).NE.1) GO TO 372                                               DD 02520
      NCON(MM9,10)=NCON(MM10,1)                                                 DD 02530
      IF (MS(KL,8).EQ.1) NCON(MM7,7)=NCON(MM9,10)                               DD 02540
372   IF (MS(K,3).NE.1) GO TO 375                                               DD 02550
      NCON(MM6,7)=NCON(MM5,4)                                                   DD 02560
      IF (MS(KL,5).EQ.1) NCON(MM8,10)=NCON(MM6,7)                               DD 02570
```

```
  375  IF ((MS(K,4).EQ.0).AND.(L3.EQ.1)) NC1=NC1+1                                    DD 02580
       IF ((((MS(K,4).EQ.0).OR.(MS(K,5).EQ.0)).AND.(L3.EQ.0))                         DD 02590
      1.AND.(LL.NE.LL2)) NC1=NC1+1                                                    DD 02600
       IF(((L3.NE.1).AND.(LL.NE.LL2)).AND.(K.EQ.KB)) NC1=NC1+1                        DD 02610
       IF (((MS(K,5).EQ.0).AND.(MS(K,6).NE.1)).AND.(LL.EQ.LL2)) NC1=NC1+1             DD 02620
  380  M1=M2+1                                                                        DD 02630
  390  K=K+1                                                                          DD 02640
  400  CONTINUE                                                                       DD 02650
  430  CONTINUE                                                                       DD 02660
       MM2=MM1                                                                        DD 02670
       MM1=M1                                                                         DD 02680
  450  CONTINUE                                                                       DD 02690
       KK=K                                                                           DD 02700
       GO TO 500                                                                      DD 02710
  470  KK=KK+NGROW                                                                    DD 02720
  500  CONTINUE                                                                       DD 02730
       NEL=M                                                                          DD 02740
       IF (NKIND.EQ.1) NJT=NCON(NEL,3)                                                DD 02750
       IF (NKIND.EQ.2) NJT=NCON(NEL,5)                                                DD 02760
       IF (NKIND.EQ.3) NJT=NCON(NEL,7)                                                DD 02770
       WRITE (6,21) NEL,NJT                                                           DD 02780
       IF (NJT.GT.MAXNJT) GO TO 800                                                   DD 02790
C      WRITE (6,20)                                                                   DD 02800
       NBAND=0                                                                        DD 02810
       DO 600 I=1,NEL                                                                 DD 02820
       NB1=NCON(I, 1)                                                                 DD 02830
       GO TO (550,560,570),NKIND                                                      DD 02840
  550  NB2=NCON(I, 3)                                                                 DD 02850
       NB3=NCON(I, 4)                                                                 DD 02860
       GO TO 580                                                                      DD 02870
  560  NB2=NCON(I, 5)                                                                 DD 02880
       NB3=NCON(I, 7)                                                                 DD 02890
       GO TO 580                                                                      DD 02900
  570  NB2=NCON(I, 7)                                                                 DD 02910
       NB3=NCON(I,10)                                                                 DD 02920
   580 CONTINUE                                                                       DD 02930
       INFB=NBAND                                                                     DD 02940
       INF1=NB1                                                                       DD 02950
       INF2=NB2                                                                       DD 02960
       INF3=NB3                                                                       DD 02970
       INFB=MAXO(INFB,(INF2-INF1))                                                    DD 02980
       INFB=MAXO(INFB,(INF3-INF1))                                                    DD 02990
       NBAND=INFB                                                                     DD 03000
       DO 590 J=1,14                                                                  DD 03010
       K=NLC(J,NKIND)                                                                 DD 03020
  590  LCON(J)=NCON(I,K)                                                              DD 03030
       WRITE (10,26) I,(LCON(J),J=1,14)                                               DD 03040
C      WRITE (6,25) I,(LCON(J),J=1,14)                                                DD 03050
```

```
  600 CONTINUE                                                          DD 03060
      NBAND=(NBAND+1)*2                                                 DD 03070
      WRITE (6,650) NBAND                                               DD 03080
  650 FORMAT (///,' HALF BAND WIDTH FOR PLISOP STIFFNESS MATRIX=',I5)   DD 03090
      RETURN                                                            DD 03100
  700 NSTOP=1                                                           DD 03110
      WRITE (6,725) MAXNEL                                              DD 03120
  725 FORMAT (///,' DATA REJECTED***NUMBER OF ELEMENTS GREATER THAN',I5 DD 03130
     1,/,' ***********************************************************')DD 03140
      RETURN                                                            DD 03150
  800 NSTOP=1                                                           DD 03160
      WRITE (6,825) MAXNJT                                              DD 03170
  825 FORMAT (///,' DATA REJECTED***NUMBER OF JOINTS GREATER THAN',I5   DD 03180
     1,/,' ***********************************************************')DD 03190
      RETURN                                                            DD 03200
      END                                                               DD 03210
```

```
      SUBROUTINE COORD                                                  EE 00010
C        THIS SUBROUTINE DETERMINES X,Y COORDINATES OF NODES FOR 4, 8,  EE 00020
C        AND 12 NODE ELEMENTS.  FORCE AND DISPLACEMENT BOUNDARY         EE 00030
C        CONDITIONS ARE ALSO DETERMINED HERE.                           EE 00040
C                                                                       EE 00050
      IMPLICIT REAL*8 (A-H,O-Z)                                         EE 00060
      DIMENSION XI(12),ETA(12)                                          EE 00070
      DIMENSION NQ1(3),NC1(4)                                           EE 00080
      COMMON/INT/NPT,NEL,NPUNCH,NSTOP,NJT,NPLOT,NKIND,NSEL,NGROW,NGCOL, EE 00090
     1           NCLOAD,NPBC                                            EE 00100
      COMMON/SF/VAL(12)                                                 EE 00110
      COMMON/MESH1/ NSCON(20,9),MS(20,8),MEL(20,4)                      EEW00120
      COMMON/MESH2/ NCR(4),NRR(4)                                       EEW00130
      COMMON/MESH3/ NCON(135,14)                                        EEW00140
      COMMON/CORD1/CORD(454,4),ELCON(1,4),NBC(454,2)                    EEW00150
      COMMON/CORD2/BOUND(48,2)                                          EEW00160
      COMMON/STAT/WR,DR,TH,W,X(33),Y(33),EXPX(48),EXPY(48)              EEW00170
      DATA NQ1/1,8,7/                                                   EE 00180
      DATA NC1/1,10,11,12/                                              EE 00190
   10 FORMAT (///,' COORDINATES OF JOINTS',//,' JOINT NUMBER',8X,'X COOREE 00200
     1DINATE',5X,'Y COORDINATE',//)                                     EE 00210
   20 FORMAT (5X,I3,2X,4(G14.5,3X),I3,2I5)                              EE 00220
   25 FORMAT (I10,2G15.8)                                               EE 00230
      F1(X1,A1)=(2.D0*(DEXP(A1*(X1+1.D0))-1.D0)/                        EE 00240
     1(DEXP(A1*2.D0)-1.D0))-1.D0                                        EE 00250
      PI=3.141592653590                                                 EE 00260
```

```
      II=1                                                              EE 00270
      K=1                                                               EE 00280
C                                                                       EE 00290
CC       GRID COLUMN LOOP                                               EE 00300
C                                                                       EE 00310
      DO 800 I=1,NGCOL                                                  EE 00320
      L1=NCR(I)                                                         EE 00330
      IF (L1.EQ.0) GO TO 790                                            EE 00340
C                                                                       EE 00350
CC       GRID ROW LOOP                                                  EE 00360
C                                                                       EE 00370
      DO 700 J=1,NGROW                                                  EE 00380
      IF (NSCON(K,1).EQ.0) GO TO 690                                    EE 00390
      M1=MEL(K,1)                                                       EE 00400
      NPTB=NSCON(K,8)                                                   EE 00410
      DELXI=2.0D0/(NKIND*NSCON(K,2))                                    EE 00420
      DELETA=2.0D0/(NKIND*NSCON(K,1))                                   EE 00430
      XIVAL=-1.0D0                                                      EE 00440
C                                                                       EE 00450
CC       SUPER ELEMENT COLUMN LOOP                                      EE 00460
C                                                                       EE 00470
      DO 600 L=1,L1                                                     EE 00480
      ETAVAL=1.0D0                                                      EE 00490
      M2=M1+NSCON(K,1)-1                                                EE 00500
C                                                                       EE 00510
CC       SUPER ELEMENT ROW LOOP                                         EE 00520
C                                                                       EE 00530
      DO 500 M=M1,M2                                                    EE 00540
      GO TO (100,150,250),NKIND                                         EE 00550
C                                                                       EE 00560
CC       LINEAR ELEMENTS                                                EE 00570
C                                                                       EE 00580
  100 DO 110 N=1,2                                                      EE 00590
  110 XI(N)=XIVAL                                                       EE 00600
      DO 120 N=3,4                                                      EE 00610
  120 XI(N)=XIVAL+  DELXI                                               EE 00620
      DO 130 N=1,4,3                                                    EE 00630
  130 ETA(N)=ETAVAL                                                     EE 00640
      DO 140 N=2,3                                                      EE 00650
  140 ETA(N)=ETAVAL-DELETA                                              EE 00660
      GO TO 350                                                         EE 00670
C                                                                       EE 00680
CC       QUADRATIC ELEMENTS                                             EE 00690
C                                                                       EE 00700
  150 DO 160 N=1,3                                                      EE 00710
  160 XI(N)=XIVAL                                                       EE 00720
      DO 170 N=4,8,4                                                    EE 00730
  170 XI(N)=XIVAL+  DELXI                                               EE 00740
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      DO 180 N=5,7                                                          EE 00750
  180 XI(N)=XIVAL+2*DELXI                                                   EE 00760
      DO 200 N=1,3                                                          EE 00770
      N1=NQ1(N)                                                             EE 00780
  200 ETA(N1)=ETAVAL                                                        EE 00790
      DO 210 N=2,6,4                                                        EE 00800
  210 ETA(N)=ETAVAL-  DELETA                                                EE 00810
      DO 220 N=3,5                                                          EE 00820
  220 ETA(N)=ETAVAL-2*DELETA                                                EE 00830
      IF((EXPX(K).NE.0.D0).OR.(EXPY(K).NE.0.D0)) GO TO 1000                 EE 00840
      GO TO 350                                                             EE 00850
C                                                                           EE 00860
C        CUBIC ELEMENTS                                                     EE 00870
C                                                                           EE 00880
  250 DO 260 N=1,4                                                          EE 00890
  260 XI(N)=XIVAL                                                           EE 00900
      DO 270 N=5,12,7                                                       EE 00910
  270 XI(N)=XIVAL+  DELXI                                                   EE 00920
      DO 280 N=6,11,5                                                       EE 00930
  280 XI(N)=XIVAL+2*DELXI                                                   EE 00940
      DO 290 N=7,10                                                         EE 00950
  290 XI(N)=XIVAL+3*DELXI                                                   EE 00960
      DO 300 N=1,4                                                          EE 00970
      N1=NC1(N)                                                             EE 00980
  300 ETA(N1)=ETAVAL                                                        EE 00990
      DO 310 N=2,9,7                                                        EE 01000
  310 ETA(N)=ETAVAL-  DELETA                                                EE 01010
      DO 320 N=3,8,5                                                        EE 01020
  320 ETA(N)=ETAVAL-2*DELETA                                                EE 01030
      DO 330 N=4,7                                                          EE 01040
  330 ETA(N)=ETAVAL-3*DELETA                                                EE 01050
      IF((EXPX(K).NE.0.D0).OR.(EXPY(K).NE.0.D0)) GO TO 1050                 EE 01060
  350 DO 400 N=1,NPT                                                        EE 01070
      N1=NCON(M,N)                                                          EE 01080
      XI1=XI(N)                                                             EE 01090
      ETA1=ETA(N)                                                           EE 01100
      CALL SHAPE (XI1,ETA1,NPTB)                                            EE 01110
      CORD(N1,1)=0.D0                                                       EE 01120
      CORD(N1,2)=0.D0                                                       EE 01130
      III=II                                                                EE 01140
      DO 360 NN=1,NPTB                                                      EE 01150
      CORD(N1,1)=CORD(N1,1)+VAL(NN)*BOUND(III,1)                            EE 01160
      CORD(N1,2)=CORD(N1,2)+VAL(NN)*BOUND(III,2)                            EE 01170
  360 III=III+1                                                             EE 01180
  400 CONTINUE                                                              EE 01190
      ETAVAL=ETAVAL-NKIND*DELETA                                            EE 01200
  500 CONTINUE                                                              EE 01210
      M1=M2+NRR(I)-NSCON(K,1)+1                                             EE 01220
```

```
      XIVAL=XIVAL+NKIND*DELXI                                                  EE 01230
  600 CONTINUE                                                                 EE 01240
      II=II+NPTB                                                               EE 01250
  690 K=K+1                                                                    EE 01260
  700 CONTINUE                                                                 EE 01270
      GO TO 800                                                                EE 01280
  790 K=K+NGROW                                                                EE 01290
  800 CONTINUE                                                                 EE 01300
C     WRITE (6,10)                                                             EE 01310
      NPBC=0                                                                   EE 01320
      NCLOAD=0                                                                 EE 01330
      READ(11,2999) I1,I2,I3,I4,I5,I6                                          EEZ01340
 2999 FORMAT(6I10)                                                             EEZ01350
      WRITE(6,2999) I1,I2,I3,I4,I5,I6                                          EEZ01360
C        CORRECT THE CIRCULAR BOUNDARY                                         EEZ01370
      DO 3000 N=I2,I3,I5                                                       EEZ01380
 3412 FORMAT (I5,D25.16)                                                       EEZ01390
 3000 CORD(N,2)=W/WR+W*DR/WR- DSQRT((W/WR)**2-CORD(N,1)**2)                    EEZ01400
      IF(NKIND.EQ.1) GO TO 73                                                  EEZ01410
      R=W/WR                                                                   EEZ01420
      DO 4000 N=I1,I4,I5                                                       EEZ01430
      IF(CORD((N+I2-I1),1).GT.(R-0.1D-5)) GO TO 58                             EEZ01440
      ANGA=DARSIN(CORD((N+I2-I1),1)/R)                                         EEZ01450
   59 IF(CORD((N-I1+1),1).EQ.0.D0) GO TO 56                                    EEZ01460
      ANGB=DARSIN(CORD((N-I2+1),1)/R)                                          EEZ01470
      IF(NKIND.EQ.3) GO TO 74                                                  EEZ01480
   57 ANGC=(ANGA+ANGB)/2.D0                                                    EEZ01490
      CORD(N,1)= DSIN(ANGC)*R                                                  EEZ01500
 4000 CORD(N,2)=W/WR+W*DR/WR-DCOS(ANGC)*R                                      EEZ01510
   73 DO 850 I=1,NJT                                                          EE 01520
      KK=0                                                                     EE 01530
C          COMPUTE DEFLECTION B. C. CODES                                      EEY01540
      IF((NPUNCH.EQ.1).AND.(CORD(I,2).EQ.0.D0))KK=1                            EEY01550
      IF((NPUNCH.EQ.2).AND.(CORD(I,2).EQ.0.D0))KK=2                            EEY01560
      IF(CORD(I,1).EQ.0.D0)KK=2                                                EEY01570
      IF((CORD(I,1).EQ.0.D0).AND.(CORD(I,2).EQ.0.D0)) KK=3                     EEY01580
      IF(I.EQ.19)KK=3                                                          EEY01590
      IF(KK.EQ.0) GO TO 5                                                      EEY01600
      NPBC=NPBC+1                                                              EEY01610
      NBC(NPBC,1)=I                                                            EEY01620
      NBC(NPBC,2)=KK                                                           EEY01630
    5 W5=5.D0*W-1.D-6                                                          EEX01640
      IF(NCLOAD.NE.0) W5=10.D0*W                                               EEX01650
      IF(CORD(I,1).GE.W5)  GO TO 801                                           EEX01660
  802 CORD(I,3)=0.D0                                                           EEX01670
      CORD(I,4)=0.D0                                                           EEX01680
C     WRITE (6,20) I,(CORD(I,J),J=1,2)                                         EEX01690
  850 CONTINUE                                                                 EEX01700
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C           COMPUTE LOAD B. C.                                          EEX01710
      IF(NPUNCH.EQ.1) GO TO 860                                         EEX01720
      IF(NPUNCH.EQ.3) GO TO 860                                         EEX01730
C           FOR APPLIED MOMENT                                          EEX01740
      NCLOAD=2                                                          EEX01750
      IF (NPUNCH.EQ.2) GO TO 861                                        EEX01760
      CORD(NJTUP,3)=-1.D6                                               EEX01770
      CORD((NJTUP+2),3)= 1.D6                                           EEX01780
      GO TO 765                                                         EEX01790
  861 CORD(NJTUP,3)=1.D6/6.D0                                           EEX01800
      CORD((NJTUP+1),3)=1.D6/3.D0                                       EEX01810
      GO TO 765                                                         EEX01820
C           FOR NO MOMENT APPLIED                                       EEX01830
  860 CORD(NJTUP,3)=1.D6/6.D0                                           EEX01840
      CORD((NJTUP+1),3)=2.D6/3.D0                                       EEX01850
      CORD((NJTUP+2),3)=1.D6/6.D0                                       EEX01860
  765 RETURN                                                            EEX01870
 1000 IF(EXPX(K).EQ.0.D0) GO TO 2000                                    EE  01880
C           FOR EXPONENT IN XI DIRECTION                                EE  01890
      DO 1001 N=1,7,2                                                   EE  01900
 1001 XI(N)=F1(XI(N),EXPX(K))                                           EE  01910
      DO 1002 N=2,6,2                                                   EE  01920
 1002 XI(N)=(XI(N-1)+XI(N+1))/2.D0                                      EE  01930
      XI(8)=(XI(7)+XI(1))/2.D0                                          EE  01940
C           FOR EXPONENT IN THE ETA DIRECTION                           EE  01950
 2000 IF(EXPY(K).EQ.0.D0) GO TO 350                                     EE  01960
      DO 2001 N=1,7,2                                                   EE  01970
 2001 ETA(N)=F1(ETA(N),EXPY(K))                                         EE  01980
      DO 2002 N=2,6,2                                                   EE  01990
 2002 ETA(N)=(ETA(N-1)+ETA(N+1))/2.D0                                   EE  02000
      ETA(8)=(ETA(7)+ETA(1))/2.D0                                       EE  02010
      GO TO 350                                                         EE  02020
 1050 IF(EXPX(K).EQ.0.D0) GO TO 2050                                    EE  02030
      DO 1051 N=1,10,3                                                  EE  02040
 1051 XI(N)=F1(XI(N),EXPX(K))                                           EE  02050
      DO 1052 N=2,11,3                                                  EE  02060
      NM=N-1                                                            EE  02070
      NP=N+2                                                            EE  02080
      IF(NP.EQ.13) NP=1                                                 EE  02090
      IF(XI(NM).LT.XI(NP)) GO TO 1053                                   EE  02100
      XI(N)=2.D0 * XI(NM)/3.D0 + XI(NP)/3.D0                            EE  02110
      XI(N+1)=2.D0 * XI(NP)/3.D0 + XI(NM)/3.D0                          EE  02120
      GO TO 1052                                                        EE  02130
 1053 XI(N)=2.D0 * XI(NP)/3.D0 + XI(NM)/3.D0                            EE  02140
      XI(NP)=2.D0 * XI(NM)/3.D0 + XI(NP)/3.D0                           EE  02150
 1052 CONTINUE                                                          EE  02160
 2050 IF(EXPY(K).EQ.0.D0) GO TO 350                                     EE  02170
      DO 2051 N=1,10,3                                                  EE  02180
```

```
2051 ETA(N)=F1(ETA(N),EXPY(K))                                          EE 02190
     DO 2052 N=2,11,3                                                   EE 02200
     NM=N-1                                                             EE 02210
     NP=N+2                                                             EE 02220
     IF(NP.EQ.13) NP=1                                                  EE 02230
     IF(ETA(NM).LT.ETA(NP)) GO TO 2053                                  EE 02240
     ETA(N)=2.D0 * ETA(NM)/3.D0 + ETA(NP)/3.D0                          EE 02250
     ETA(N+1)=2.D0 * ETA(NP)/3.D0 + ETA(NM)/3.D0                        EE 02260
     GO TO 2052                                                         EE 02270
2053 ETA(N)=2.D0 * ETA(NP)/3.D0 + ETA(NM)/3.D0                          EE 02280
     ETA(N+1)=2.D0 * ETA(NM)/3.D0 + ETA(NP)/3.D0                        EE 02290
2052 CONTINUE                                                           EE 02300
     GO TO 350                                                          EE 02310
  74 ANGC=2.D0 * ANGB/3.D0 + ANGA/3.D0                                  EEZ02320
     ANGD=2.D0 * ANGA/3.D0 + ANGB/3.D0                                  EEZ02330
     CORD(N,1)=DSIN(ANGC) * R                                           EEZ02340
     CORD(N,2)=W/WR + W * DR/WR - DCOS(ANGC) * R                        EEZ02350
     CORD(N+16,1)=DSIN(ANGD)* R                                         EEZ02360
     CORD(N+16,2)=W/WR + W * DR/WR - DCOS(ANGC)*R                       EEZ02370
     GO TO 73                                                           EEZ02380
 801 NCLOAD=NJT-I+1                                                     EEX02390
     NJTUP=I                                                            EEX02400
     GO TO 802                                                          EEX02410
  56 ANGB=0.D0                                                          EEZ02420
     GO TO 57                                                           EEZ02430
  58 ANGA=PI/2.D0                                                       EEZ02440
     GO TO 59                                                           EEZ02450
     END                                                                EE 02460
```

```
     SUBROUTINE SHAPE(X,Y,NPEL)                                         FF 00010
     IMPLICIT REAL*8 (A-H,O-Z)                                          FF 00020
     DIMENSION XYL(4,2),XYQ(8,2),XYC(12,2),IPERM(4)                     FF 00030
     COMMON/SF/VAL(12)                                                  FF 00040
     DATA XYL/1.0D0,-1.0D0,-1.0D0,1.0D0,1.0D0,1.0D0,-1.0D0,-1.0D0/      FF 00050
     DATA XYQ/1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0,1.0D0,1.0D0,1.0D0,FF 00060
    11.0D0,1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0/                      FF 00070
     DATA XYC/1.0D0,.33333333333333D0,-.33333333333333D0,-1.0D0,-1.0DFF 00080
    10,-1.0D0,-1.0D0,-.33333333333333D0,.33333333333333D0,1.0D0,1.0D0FF 00090
    2,1.0D0,1.0D0,1.0D0,1.0D0,1.0D0,.33333333333333D0,-.33333333333333FF 00100
    33D0,-1.0D0,-1.0D0,-1.0D0,-1.0D0,-.33333333333333D0,.33333333333333FF 00110
    433D0/                                                              FF 00120
     FL(X,Y,X1,Y1)=(ONE+X*X1)*(ONE+Y*Y1)/FOUR                           FF 00130
     FQC(X,Y,X1,Y1)=(ONE+X*X1)*(ONE+Y*Y1)*(X*X1+Y*Y1-ONE)/FOUR          FF 00140
     FQM(X,Y,Y1)=(ONE-X*X)*(ONE+Y*Y1)/TWO                               FF 00150
```

```
      FCC(X,Y,X1,Y1)=(ONE+X*X1)*(ONE+Y*Y1)*(.9D1*(X*X+Y*Y)-.1D2)/3.2D1   FF 00160
      FCM(X,Y,X1,Y1)=(ONE-X*X)*(ONE+.9D1*X*X1)*(ONE+Y*Y1)*.9D1/3.2D1     FF 00170
      ONE=1.0D0                                                          FF 00180
      TWO=2.0D0                                                          FF 00190
      FOUR=4.0D0                                                         FF 00200
      IGO=NPEL/4                                                         FF 00210
      GO TO (50,150,450),IGO                                             FF 00220
   50 CONTINUE                                                           FF 00230
C        LINEAR FUNCTIONS                                                FF 00240
      DO 100 I=1,4                                                       FF 00250
      X1=XYL(I,1)                                                        FF 00260
      Y1=XYL(I,2)                                                        FF 00270
  100 VAL(I)=FL(X,Y,X1,Y1)                                               FF 00280
      RETURN                                                             FF 00290
  150 CONTINUE                                                           FF 00300
C        QUADRATIC FUNCTIONS                                             FF 00310
      DO 200 I=1,7,2                                                     FF 00320
      X1=XYQ(I,1)                                                        FF 00330
      Y1=XYQ(I,2)                                                        FF 00340
  200 VAL(I)=FQC(X,Y,X1,Y1)                                              FF 00350
      DO 300 I=2,6,4                                                     FF 00360
      Y1=XYQ(I,2)                                                        FF 00370
  300 VAL(I)=FQM(X,Y,Y1)                                                 FF 00380
      DO 400 I=4,8,4                                                     FF 00390
      X1=XYQ(I,1)                                                        FF 00400
  400 VAL(I)=FQM(Y,X,X1)                                                 FF 00410
      RETURN                                                             FF 00420
  450 CONTINUE                                                           FF 00430
C        CUBIC FUNCTIONS                                                 FF 00440
      DO 500 I=1,10,3                                                    FF 00450
      X1=XYC(I,1)                                                        FF 00460
      Y1=XYC(I,2)                                                        FF 00470
  500 VAL(I)=FCC(X,Y,X1,Y1)                                              FF 00480
      IPERM(1)=2                                                         FF 00490
      IPERM(2)=3                                                         FF 00500
      IPERM(3)=8                                                         FF 00510
      IPERM(4)=9                                                         FF 00520
      DO 600 I=1,4                                                       FF 00530
      IJ=IPERM(I)                                                        FF 00540
      X1=XYC(IJ,1)                                                       FF 00550
      Y1=XYC(IJ,2)                                                       FF 00560
  600 VAL(IJ)=FCM(X,Y,X1,Y1)                                             FF 00570
      IPERM(1)=5                                                         FF 00580
      IPERM(2)=6                                                         FF 00590
      IPERM(3)=11                                                        FF 00600
      IPERM(4)=12                                                        FF 00610
      DO 700 I=1,4                                                       FF 00620
      IJ=IPERM(I)                                                        FF 00630
```

```
      X1=XYC(IJ,1)                                                      FF 00640
      Y1=XYC(IJ,2)                                                      FF 00650
  700 VAL(IJ)=FCM(Y,X,Y1,X1)                                            FF 00660
      RETURN                                                            FF 00670
      END                                                               FF 00680



      SUBROUTINE GRID                                                   GG 00010
      IMPLICIT REAL*8 (A-H,O-Z)                                         GG 00020
      REAL*4 X,Y,RANGE,XSCALE,YSCALE,LABEL                              GG 00030
      DATA LABEL/4H    /                                                GG 00040
      INTEGER*4 N1,MC,ITYPE,IXUP,IYRT,MDXAX,MDYAX,IWIDE,IHIGH,IGRID,    GG 00050
     1LAST,NNJT                                                         GG 00060
      DIMENSION X(454), Y(454), RANGE(4)                                GGW00070
      COMMON/MESH1/ NSCON(20,9),MS(20,8),MEL(20,4)                      GGW00080
      COMMON/MESH2/ NCR(4),NRR(4)                                       GGW00090
      COMMON/MESH3/ NCON(135,14)                                        GGW00100
      COMMON/CORD1/CORD(454,4),ELCON(1,4),NBC(454,2)                    GGW00110
      COMMON/TITL/TITLE(12)                                             GG 00120
      COMMON/INT/NPT,NEL,NPUNCH,NSTOP,NJT,NPLOT,NKIND,NSEL,NGROW,NGCOL, GG 00130
     1           NCLOAD,NPBC                                            GG 00140
C                                                                       GG 00150
C        X,Y DIMENSIONED MAXNJT                                         GG 00160
C                                                                       GG 00170
      DATA ITYPE/0/ ,IXUP/15/,IYRT/0/ ,MDXAX/2/ ,MDYAX/2/ ,IWIDE/9/ ,   GG 00180
     1IHIGH/15/ ,IGRID/0/                                               GG 00190
      XMAX=-1.0D+20                                                     GG 00200
      YMAX=-1.0D+20                                                     GG 00210
      XMIN= 1.0D+20                                                     GG 00220
      YMIN= 1.0D+20                                                     GG 00230
      DO 20 I=1,NJT                                                     GG 00240
      XMAX=DMAX1(XMAX,CORD(I,1))                                        GG 00250
      YMAX=DMAX1(YMAX,CORD(I,2))                                        GG 00260
      XMIN=DMIN1(XMIN,CORD(I,1))                                        GG 00270
   20 YMIN=DMIN1(YMIN,CORD(I,2))                                        GG 00280
      IF (XMIN.GE.0.D0) GO TO 40                                        GG 00290
      XMAX=XMAX-XMIN                                                    GG 00300
      DO 30 I=1,NJT                                                     GG 00310
   30 CORD(I,1)=CORD(I,1)-XMIN                                          GG 00320
   40 IF (YMIN.GE.0.D0) GO TO 60                                        GG 00330
      YMAX=YMAX-YMIN                                                    GG 00340
      DO 50 I=1,NJT                                                     GG 00350
   50 CORD(I,2)=CORD(I,2)-YMIN                                          GG 00360
   60 GO TO (70,110),NPLOT                                              GG 00370
C                                                                       GG 00380
```

```
C          PLOT ON PRINTER                                                    GG 00390
C                                                                             GG 00400
 70    RANGE(1)=XMAX                                                          GG 00410
       RANGE(2)=0.0                                                           GG 00420
       RANGE(3)=YMAX                                                          GG 00430
       RANGE(4)=0.0                                                           GG 00440
       IF (RANGE(1).LT.RANGE(3)) RANGE(1)=RANGE(3)                            GG 00450
       IF (RANGE(1).GT.RANGE(3)) RANGE(3)=RANGE(1)                            GG 00460
       DO 80 I=1,NJT                                                          GG 00470
       X(I)=CORD(I,1)                                                         GG 00480
 80    Y(I)=CORD(I,2)                                                         GG 00490
       WRITE (6,90)                                                           GG 00500
 90    FORMAT ('1')                                                           GG 00510
       NNJT=NJT                                                               GG 00520
       CALL UTPLOT (X,Y,NNJT,RANGE,1,0)                                       GG 00530
       RETURN                                                                 GG 00540
C                                                                             GG 00550
C          PLOT ON PLOTTER                                                    GG 00560
C                                                                             GG 00570
 110   XSCALE=1.5D0*(YMAX/9.D0)                                               GG 00580
       YSCALE=1.5D0*(XMAX/15.D0)                                              GG 00590
       IF (XSCALE.GT.YSCALE) YSCALE=XSCALE                                    GG 00600
       IF (XSCALE.LT.YSCALE) XSCALE=YSCALE                                    GG 00610
       N1=NPT+1                                                               GG 00620
       MC=1                                                                   GG 00630
       DO 150 I=1,NEL                                                         GG 00640
       IF (I.EQ.NEL) MC=3                                                     GG 00650
       DO 120 J=1,NPT                                                         GG 00660
       J1=NCON(I,J)                                                           GG 00670
       IF (J.EQ.1) J2=J1                                                      GG 00680
       X(J)=CORD(J1,2)                                                        GG 00690
 120   Y(J)=-CORD(J1,1)                                                       GG 00700
       J=J+1                                                                  GG 00710
       X(J)=CORD(J2,2)                                                        GG 00720
       Y(J)=-CORD(J2,1)                                                       GG 00730
       CALL DRAW (N1,X ,Y ,MC,ITYPE,LABEL,TITLE,XSCALE,YSCALE,IXUP,IYRT,      GG 00740
      1MDXAX,MDYAX,IWIDE,IHIGH,IGRID,LAST)                                    GG 00750
       MC=2                                                                   GG 00760
 150   CONTINUE                                                               GG 00770
 200   CONTINUE                                                               GG 00780
       RETURN                                                                 GG 00790
       END                                                                    GG 00800



       SUBROUTINE PLISOP                                                      HH 00010
C    THIS PROGRAM PERFORMS A PLANE STRESS OR A PLANE STRAIN ANALYSIS          HH 00020
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C   BY THE FINITE ELEMENT METHOD. NUMERICALLY INTEGRATED ISOPARAMETRIC  HH 00030
C   ELEMENTS ARE USED THROUGHOUT.                                       HH 00040
C   PROFESSOR GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL  1972      HH 00050
C    MODIFIED BY L.J.SMITH, MAY 1974                                    HH 00060
C                                                                       HH 00070
C  INPUT CARDS NEEDED (PASSED VIA MAIN)                                 HH 00080
C                                                                       HH 00090
C TITLE                        (20A4)          ONE CARD PER PROBLEM     HH 00100
C                                                                       HH 00110
C  PROBLEM PARAMETERS          (10I5)          ONE CARD PER PROBLEM     HH 00120
C   COL.   VARIABLE                                                     HH 00130
C    1-5       NEL  NUMBER OF ELEMENTS (MAX. IS 135, 8 NODE ELEMENTS)   HH 00140
C    6-10      NJT  NUMBER OF JOINTS (MAX. IS 454)                      HH 00150
C   11-15      NMAT  NUMBER OF MATERIALS (MAX. IS 1)                    HH 00160
C   16-20      NCLOAD  NUMBER OF CONCENTRATED LOADS                     HH 00170
C   21-25      NPBC  NUMBER OF JOINTS WITH BOUNDARY CONDITIONS          HH 00180
C   26-30      NSTRESS  ZERO FOR PLANE STRESS PROBLEMS, ONE FOR P.STRAIN HH 00190
C   31-35      NGP  NUMBER OF GAUSS POINTS DESIRED IN THE INTEGRATION   HH 00200
C   36-40      NTLD  1  IF A THERMAL LOAD IS PRESENT  0  OTHERWISE      HH 00210
C   41-45      NTBY  1  IF A BODY FORCE LOAD IS PRESENT  0 OTHERWISE    HH 00220
C   46-50      NTIN  1  IF AN INITIAL STRETCH IS TO BE CONSIDERED       HH 00230
C              IF NTLD, NTBY OR NTIN ARE DIFFERENT THAN 0, THEN NJT     HH 00240
C              CARDS WILL BE NEEDED BELOW FOR THE CONSISTENT LOAD VECTOR HH 00250
C                                                                       HH 00260
C  JOINT CARDS              (I10,4G15.8,1I5)  NJT CARDS PER PROBLEM     HH 00270
C   COL.   VARIABLE                                                     HH 00280
C    1-10      JOINT NUMBER                                             HH 00290
C   11-25      X  COORDINATE OF THE JOINT                               HH 00300
C   26-40      Y  COORDINATE OF THE JOINT                               HH 00310
C   41-55      X  COMPONENT OF THE LOAD AT THE JOINT                    HH 00320
C   56-70      Y  COMPONENT OF THE LOAD AT THE JOINT                    HH 00330
C   71-75      IND  IF IND IS NOT  0  THE COORDINATE SYSTEM IS POLAR,   HH 00340
C              X  IS THE RADIUS VECTOR, Y  IS THE POLAR ANGLE IN DEGREES HH 00350
C              FOR THE LOAD, X  IS A RADIAL LOAD AND Y  IS A TANGENT LOADHH 00360
C                                                                       HH 00370
C  MATERIAL PROPERTIES         (CODED IN INPUT)                         HH 00380
C   COL.   VARIABLE                                                     HH 00390
C    1-10      MATERIAL NUMBER                                          HH 00400
C   11-20      YOUNG'S MODULUS                                          HH 00410
C   21-30      POISSON'S RATIO                                          HH 00420
C   31-40      COEFFICIENT OF THERMAL EXPANSION                         HH 00430
C   41-50      THICKNESS OF THE ELEMENT                                 HH 00440
C                                                                       HH 00450
C  ELEMENT CARDS               (15I4)          NEL  CARDS PER PROBLEM   HH 00460
C   COL.   VARIABLE                                                     HH 00470
C    1-4       ELEMENT IDENTIFICATION NUMBER                            HH 00480
C    5-52      ELEMENT CONNECTIVITY (COUNTERCLOCKWISE)                  HH 00490
C   53-56      KIND  1  FOR FOUR NODAL POINT ELEMENT                    HH 00500
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C                    2  FOR EIGHT NODAL POINT ELEMENT                          HH 00510
C                    3  FOR TWELVE NODAL POINT ELEMENT                         HH 00520
C     57-60      TYPE  THIS IS THE MATERIAL IDENTIFICATION NUMBER               HH 00530
C                                                                               HH 00540
C   BOUNDARY CONDITION CARDS  (2I5)          NPBC  CARDS PER PROBLEM           HH 00550
C    COL.   VARIABLE                                                            HH 00560
C     1-5  JOINT NUMBER                                                         HH 00570
C     6-10 CODE  1 FOR A ROLLER ON AN X AXIS, DISP. IN Y DIRECT. IS ZERO        HH 00580
C                2 FOR A ROLLER ON AN Y AXIS, DISP. IN X DIRECT. IS ZERO        HH 00590
C                3 FOR A FIXED JOINT, BOTH DISP. COMPONENTS ARE ZERO            HH 00600
C                4 FOR A ROLLER ON AN X AXIS, THE DISP. IN THE Y DIRECT.        HH 00610
C                  IS SPECIFIED ON THE JOINT CARD IN COL. 41-50                 HH 00620
C                5 FOR A ROLLER ON A Y AXIS. THE DISP. IN THE X DIRECT.         HH 00630
C                  IS SPECIFIED ON THE JOINT CARD IN COL. 31-40                 HH 00640
C                6 FIXED JOINT WITH DISP. SPECIFIED IN BOTH X AND Y             HH 00650
C                  DIRECT. ON THE JOINT CARD IN COL.31-40 AND COL.41-50         HH 00660
C                7 FOR A ROLLER ON AN AXIS INCLINED ALPHA DEGREES               HH 00670
C                  WITH RESPECT TO THE X AXIS. THE JOINT CARD MAY               HH 00680
C                  CONTAIN A TANGENTIAL LOAD IN COL. 31-40 AND THE              HH 00690
C                  ANGLE ALPHA DEGREES IN COL. 41-50                            HH 00700
C                                                                               HH 00710
C   CONSISTENT LOAD VECTOR (I10,2G25.16)    NJT CARDS PER PROBLEM               HH 00720
C    NO CARDS NEEDED IF THERE ARE NO DISTRIBUTED LOADS                          HH 00730
C    COL.   VARIABLE                                                            HH 00740
C     1-10 JOINT NUMBER                                                         HH 00750
C    11-35 X COMPONENT OF LOAD                                                  HH 00760
C    36-70 Y COMPONENT OF LOAD                                                  HH 00770
C                                                                               HH 00780
      IMPLICIT REAL*8 (A-H,O-Z)                                                 HH 00790
      COMMON/MDIM/MEL,MJT,MMT,MBD                                               HH 00800
      COMMON/TITL/TITLE(12)                                                     HH 00810
      COMMON/SOL/BGK( 908,64),ALOAD( 908),ABGN                                  HHW00820
      COMMON/UNIT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(135,14),NSTRES,NGP,            HHW00830
     1LM(8),LJT(8),NTLD,LTBY,NTIN                                               HHW00840
      COMMON/CORD1/COORD(454,2),CLOAD(454,2),ELCON(1,4),NBC(454,2)              HHW00850
      DIMENSION REACT(455,2),NCODE(454)                                         HHW00860
      EQUIVALENCE (BGK(1,1),REACT(1,1))                                         HHW00870
      MEL=135                                                                   HHW00880
      MJT=454                                                                   HHW00890
      MMT=1                                                                     HHW00900
      MBD=64                                                                    HHW00910
      PI=3.141592653590                                                         HH 00920
      ZRO=0.000                                                                 HH 00930
   10 NBAND=0                                                                   HH 00940
      DO 20 I=1,MJT                                                             HH 00950
   20 NCODE(I)=0                                                                HH 00960
      CALL INPUT                                                                HH 00970
      NEQ=2*NJT                                                                 HH 00980
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      NPEL=NCON(1,13)*4                                                 HH 00990
      DO 300 I=1,NEL                                                    HH 01000
      DO 100 J=1,NPEL                                                   HH 01010
  100 LM(J)=NCON(I,J)                                                   HH 01020
      NPELM=NPEL-1                                                      HH 01030
      DO 200 K=1,NPELM                                                  HH 01040
      JK=K+1                                                            HH 01050
      DO 200 L=JK,NPEL                                                  HH 01060
  200 NBAND=MAXO(NBAND,IABS(LM(K)-LM(L)))                               HH 01070
  300 CONTINUE                                                          HH 01080
      NBAND=(NBAND+1)*2                                                 HH 01090
      IF(NBAND.GT.MBD)GO TO 6000                                        HH 01100
      DO 350 I=1,NEQ                                                    HH 01110
      ALOAD(I)=ZRO                                                      HH 01120
      DO 350 J=1,NBAND                                                  HH 01130
  350 BGK(I,J)=ZRO                                                      HH 01140
      CALL MERGE(NPEL,NEQ)                                              HH 01150
      CALL BCOND(NBAND)                                                 HH 01160
      CALL LDLT(NEQ,NBAND)                                              HH 01170
      DO 500 I=1,NPBC                                                   HH 01180
      IJT=NBC(I,1)                                                      HH 01190
  500 NCODE(IJT)=NBC(I,2)                                               HH 01200
      DO 600 I=1,NJT                                                    HH 01210
      II=I*2-1                                                          HH 01220
      KODE=NCODE(I)                                                     HH 01230
      IF(KODE.EQ.7) GO TO 560                                           HH 01240
      IF((KODE.EQ.5).OR.(KODE.EQ.6)) GO TO 550                          HH 01250
      ALOAD(II)=CLOAD(I,1)+ALOAD(II)                                    HH 01260
  550 II=II+1                                                           HH 01270
      IF((KODE.EQ.4).OR.(KODE.EQ.6)) GO TO 600                          HH 01280
      ALOAD(II)=CLOAD(I,2)+ALOAD(II)                                    HH 01290
      GO TO 600                                                         HH 01300
  560 ALF=CLOAD(I,2)*PI/180.0D0                                         HH 01310
      COSA=DCOS(ALF)                                                    HH 01320
      SINA=DSIN(ALF)                                                    HH 01330
      IIP=II+1                                                          HH 01340
      A1=ALOAD(II)*COSA+ALOAD(IIP)*SINA+CLOAD(I,1)                      HH 01350
      A2=ALOAD(IIP)*COSA-ALOAD(II)*SINA                                 HH 01360
      ALOAD(II)=A1                                                      HH 01370
      ALOAD(IIP)=A2                                                     HH 01380
  600 CONTINUE                                                          HH 01390
      CALL SOLV(NEQ,NBAND)                                              HH 01400
      DO 630 I=1,NPBC                                                   HH 01410
      DO 630 J=1,2                                                      HH 01420
  630 REACT(I,J)=ZRO                                                    HH 01430
      CALL DISPL                                                        HH 01440
      WRITE(6,1000)                                                     HH 01450
      DO 700 I=1,NJT                                                    HH 01460
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      II=I*2-1                                                          HH 01470
      III=II+1                                                          HH 01480
      WRITE(6,2000) I,ALOAD(II),ALOAD(III)                              HH 01490
  700 CONTINUE                                                          HH 01500
 2000 FORMAT(2X,I3,2G15.4)                                              HH 01510
 1000 FORMAT(//,' DISPLACEMENTS',//,3X,' JOINT',1X,'  U',12X,'  V',/)   HH 01520
      WRITE(6,3000)                                                     HH 01530
 3000 FORMAT(//,'     REACTIONS',//,3X,' JOINT',1X,' RX',12X,' RY',/)   HH 01540
      DO 800 I=1,NPBC                                                   HH 01550
      WRITE(6,2000) NBC(I,1),REACT(I,1),REACT(I,2)                      HH 01560
  800 CONTINUE                                                          HH 01570
      WRITE(6,4000)                                                     HH 01580
 4000 FORMAT(/,' EQUILIBRIUM CHECK ',/)                                 HH 01590
      MJP=MJT+1                                                         HH 01600
      WRITE(6,2500) REACT(MJP,1),REACT(MJP,2)                           HH 01610
 2500 FORMAT(5X,2G15.4)                                                 HH 01620
      CALL STRESS(NPEL)                                                 HH 01630
      RETURN                                                            HH 01640
 6000 WRITE(6,6500) MBD,NBAND                                           HH 01650
 6500 FORMAT(5X,' MAX. BAND (',1I3,') EXCEEDED, NBAND = ',1I4)          HH 01660
      STOP                                                              HH 01670
      END                                                               HH 01680
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```
      SUBROUTINE INPUT                                                  II 00010
      IMPLICIT REAL*8 (A-H,O-Z)                                         II 00020
      COMMON/MDIM/MEL,MJT,MMT,MBD                                       II 00030
      COMMON/TITL/TITLE(12)                                             II 00040
      COMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(135,14),NSTRES,NGP,     IIW00050
     1LM(8),LJT(8),NTLD,LTBY,NTIN                                       IIW00060
      COMMON/CORD1/COARD(454,2),CLOAD(454,2),ELCON(1,4),NBC(454,2)      IIW00070
      DATA CHK /'STOP    '/                                             II 00080
      ZRO=0.0D0                                                         II 00090
      PI=3.1415926535D0                                                 II 00100
      REWIND 10                                                         II 00110
 1000 FORMAT (10A8)                                                     II 00120
      IF (TITLE(1).EQ.CHK) STOP                                         II 00130
      DO 1050 I=1,MJT                                                   II 00140
      DO 1050 J=1,2                                                     II 00150
C     NBC(I,J)=0                                                        II 00160
C     COARD(I,J)=ZRO                                                    II 00170
C     CLOAD(I,J)=ZRO                                                    II 00180
 1050 CONTINUE                                                          II 00190
      DO 1060 I=1,MMT                                                   II 00200
      DO 1060 J=1,4                                                     II 00210
C     ELCON(I,J)=ZRO                                                    II 00220
 1060 CONTINUE                                                          II 00230
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      DO 1070 I=1,MEL                                                    II 00240
      DO 1070 J=1,14                                                     II 00250
C     NCON(I,J)=0                                                        II 00260
 1070 CONTINUE                                                           II 00270
C     WRITE(6,1000) TITLE                                                II 00280
C                                                                        II 00290
C     READ STRUCTURE PARAMETERS                                          II 00300
C                                                                        II 00310
 1100 FORMAT(10I5)                                                       II 00320
      READ(11,1100) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN   II 00330
      IF(NEL.GT.MEL) GO TO 4300                                          II 00340
      IF(NJT.GT.MJT) GO TO 4400                                          II 00350
      IF(NMAT.GT.MMT) GO TO 4500                                         II 00360
      IF(NCLOAD.GT.MJT) GO TO 4600                                       II 00370
      IF(NPBC.GT.MJT) GO TO 4700                                         II 00380
      WRITE (6,1200) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP                 II 00390
 1200 FORMAT (//,' NUMBER OF ELEMENTS=',I5,//,' TOTAL NUMBER OF JOINTS='II 00400
     1,I5,//,' NUMBER OF MATERIALS=',I5,//,' NUMBER OF CONCENTRATED LOADII 00410
     2S=',I5,//,' NUMBER OF JOINTS WITH BOUNDARY CONDITIONS=',I5,//,' THII 00420
     3E VALUE OF NSTRES=',I5,//,' NUMBER OF GAUSS POINTS USED ',I5)      II 00430
C                                                                        II 00440
C     WRITE COORDINATES OF JOINTS AND CONCENTRATED LOADS                 II 00450
C                                                                        II 00460
      WRITE (6,1300)                                                     II 00470
 1300 FORMAT (///,' JOINT NUMBER',5X,'X COORDINATE',5X,'Y COORDINATE',7XII 00480
     1,'X LOAD',9X,' Y LOAD'//)                                          II 00490
      DO 1600 I=1,NJT                                                    II 00500
      WRITE (6,1700) I,COARD(I,1),COARD(I,2),CLOAD(I,1),CLOAD(I,2)       II 00510
 1600 CONTINUE                                                           II 00520
 1700 FORMAT     (5X,I3,10X,G14.5,3X,G14.5,5X,G14.5,3X,G14.5)            II 00530
C                                                                        II 00540
C     WRITE MATERIAL PROPERTIES                                          II 00550
C                                                                        II 00560
      WRITE(6,2000)                                                      II 00570
 2000 FORMAT(///,' MATERIAL PROPERTIES',//,'MAT. NO. YOUNG,S MOD. PO     II 00580
     1ISSON,S RET.      ALPHA          THICKNESS',//)                   II 00590
      I=1                                                                II 00600
      ELCON(I,1)=3.D7                                                    II 00610
      ELCON(I,2)=3.D-1                                                   II 00620
      ELCON(I,3)=0.D0                                                    II 00630
  100 ELCON(I,4)=0.D0                                                    II 00640
      WRITE(6,2200) I,(ELCON(I,J),J=1,4)                                 II 00650
 2200 FORMAT(16X,I3,1X,4G15.6)                                           II 00660
C                                                                        II 00670
C     READ CONNECTIVITY MATRIX                                           II 00680
C                                                                        II 00690
      WRITE (6,2300)                                                     II 00700
 2300 FORMAT (///,' CONNECTIVITY MATRIX',//,'   EL NO',59X,' KIND',' TYPII 00710
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```
     1E'//)                                                                    II 00720
      DO 2400 I=1,NEL                                                          II 00730
 2400 READ(10,2500) IJT,(NCON(IJT,J),J=1,14)                                   II 00740
 2500 FORMAT(15I4)                                                             II 00750
      DO 2600 I=1,NEL                                                          II 00760
      WRITE (6,2700) I,(NCON(I,J),J=1,14)                                      II 00770
 2600 CONTINUE                                                                 II 00780
 2700 FORMAT(2X,I3,14I5)                                                       II 00790
C                                                                              II 00800
CC    WRITE BOUNDARY CONDITION CODES                                           II 00810
C                                                                              II 00820
      WRITE (6,2800)                                                           II 00830
 2800 FORMAT (///,' BOUNDARY CONDITION CODE',//,5X,'JNT NO',6X,'CODE'//)       II 00840
      DO 2900 I=1,NPBC                                                         II 00850
      WRITE (6,3100) (NBC(I,J),J=1,2)                                          II 00860
 2900 CONTINUE                                                                 II 00870
 3100 FORMAT(5X,I5,I10)                                                        II 00880
      NPBCM=NPBC-1                                                             II 00890
      NPBCT=(NPBC/2)+1                                                         II 00900
      DO 4000 K=1,NPBCT                                                        II 00910
      DO 4000 I=1,NPBCM                                                        II 00920
      IF(NBC(I,2).EQ.7) GO TO 3600                                             II 00930
      GO TO 4000                                                               II 00940
 3600 N1=NBC(I,1)                                                              II 00950
      DO 3800 J=I,NPBCM                                                        II 00960
      NBC(J,1)=NBC(J+1,1)                                                      II 00970
 3800 NBC(J,2)=NBC(J+1,2)                                                      II 00980
      NBC(NPBC,1)=N1                                                           II 00990
      NBC(NPBC,2)=7                                                            II 01000
 4000 CONTINUE                                                                 II 01010
C                                                                              II 01020
CC    READ LOAD VECTOR  FROM SERVICE PROGRAM                                   II 01030
C                                                                              II 01040
      IF((NTLD+NTBY+NTIN).EQ.0) RETURN                                         II 01050
      DO 4200 I=1,NJT                                                          II 01060
      READ(5,4100) IJT,CLX,CLY                                                 II 01070
 4100 FORMAT(5X,1I5,2G25.16)                                                   II 01080
      CLOAD(IJT,1)=CLOAD(IJT,1)+CLX                                            II 01090
      CLOAD(IJT,2)=CLOAD(IJT,2)+CLY                                            II 01100
 4200 CONTINUE                                                                 II 01110
      RETURN                                                                   II 01120
 4300 WRITE(6,4350) MEL,NEL                                                    II 01130
 4350 FORMAT(5X,' MAX. NUMBER (',1I3,') OF ELEMENTS EXCEEDED, NEL=',1I4)       II 01140
      STOP                                                                     II 01150
 4400 WRITE(6,4450) MJT,NJT                                                    II 01160
 4450 FORMAT(5X,' MAX. NUMB. (',1I3,') OF JOINTS EXCEEDED, NJT =',1I4)         II 01170
      STOP                                                                     II 01180
 4500 WRITE(6,4550) MMT,NMAT                                                   II 01190
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4550 FORMAT(5X,' MAX. NUMB. (',I12,') OF MATERIAL EXCEEDED, NMAT=',I14)II 01200
     STOP                                                              II 01210
4600 WRITE(6,4650) MJT,NCLOAD                                          II 01220
4650 FORMAT(5X,' MAX. NUMB. (',I13,') OF C. LOADS EXC., NCLOAD =',I14) II 01230
     STOP                                                              II 01240
4700 WRITE(6,4750) MJT,NPBC                                            II 01250
4750 FORMAT(5X,' MAX. N. (',I13,') OF JOINTS WITH BOUND. COND. EXCDED, II 01260
    1 NPBC =',I14)                                                     II 01270
     STOP                                                              II 01280
     END                                                               II 01290


     SUBROUTINE MERGE(NPEL,NEQ)                                        JJ 00010
     IMPLICIT REAL*8 (A-H,O-Z)                                         JJ 00020
     DIMENSION          STK(16,16),AK(16,16),B(3,16)                   JJW00030
     COMMON/SOL/BGK( 908,64),ALOAD( 908),ABGN                          JJW00040
     COMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(135,14),NSTRES,NGP,     JJW00050
    1LM(8),LJT(8),NTLD,LTBY,NTIN                                       JJW00060
     COMMON/CORD1/COARD(454,2),CLOAD(454,2),ELCON(1,4),NBC(454,2)      JJW00070
     COMMON COORD(12,2),ELAST(3,3),SS(24,16),SN(24,16)                 JJW00080
     COMMON/TITL/TITLE(12)                                             JJ 00090
     REWIND 12                                                         JJ 00100
     ZRO=0.0D0                                                         JJ 00110
      DO 100 I=1,36                                                    JJ 00120
     DO 100 J=1,24                                                     JJ 00130
     SS(I,J)=ZRO                                                       JJ 00140
 100 SN(I,J)=ZRO                                                       JJ 00150
     DO 200 IK=1,NEL                                                   JJ 00160
     DO 110 NN=1,NPEL                                                  JJ 00170
     LJT(NN)=NCON(IK,NN)                                               JJ 00180
 110 LM(NN)=(NCON(IK,NN)-1)*2                                          JJ 00190
     NS=3*NPEL                                                         JJ 00200
     DO 120 I1=1,NPEL                                                  JJ 00210
     I2=LJT(I1)                                                        JJ 00220
     DO 120 J1=1,2                                                     JJ 00230
 120 COORD(I1,J1)=COARD(I2,J1)                                         JJ 00240
     MATN=NCON(IK,14)                                                  JJ 00250
     N=NCON(IK,13)*8                                                   JJ 00260
     YM=ELCON(MATN,1)                                                  JJ 00270
     PRTIO=ELCON(MATN,2)                                               JJ 00280
     TK=ELCON(MATN,4)                                                  JJ 00290
     IF(NSTRES.EQ.1) TK=1.0D0                                          JJ 00300
     CALL ELASM(ELAST,YM,PRTIO,NSTRES)                                 JJ 00310
     CALL QUADS(STK,AK,B,NS,N,NGP)                                     JJ 00320
     WRITE(12) SS,SN                                                   JJ 00330
     IF(TK.EQ.0.0D0) TK=1.0D0                                          JJ 00340
     DO 200 I=1,NPEL                                                   JJ 00350
```

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      DO 200 J=1,NPEL                                                    JJ 00360
      DO 200 K=1,2                                                       JJ 00370
      II=LM(I)+K                                                         JJ 00380
      KK=2*(I-1)+K                                                       JJ 00390
      DO 200 L=1,2                                                       JJ 00400
      JJ=LM(J)+L-II+1                                                    JJ 00410
      IF(JJ.LE.0) GO TO 200                                              JJ 00420
      LL=2*(J-1)+L                                                       JJ 00430
      BGK(II,JJ)=BGK(II,JJ)+STK(KK,LL)*TK                                JJ 00440
  200 CONTINUE                                                           JJ 00450
      SUM=ZRO                                                            JJ 00460
      CON=ZRO                                                            JJ 00470
      DO 300 I=1,NEQ                                                     JJ 00480
  300 SUM=SUM+BGK(I,1)                                                   JJ 00490
      ABGN=SUM*1.0D20                                                    JJ 00500
      RETURN                                                             JJ 00510
      END                                                                JJ 00520


      SUBROUTINE ELASM(D,E,PR,NN)                                        KK 00010
      IMPLICIT REAL*8 (A-H,O-Z)                                          KK 00020
C     THIS SUBROUTINE ASSEMBLYS THE ELASTIC MATRIX FOR PLANE             KK 00030
C     STRAIN (NN=1) OR PLANE STRESS (NN=0)                               KK 00040
      DIMENSION D(3,3)                                                   KK 00050
      ON=1.0D0                                                           KK 00060
      TW=2.0D0                                                           KK 00070
      DO 20 I=1,3                                                        KK 00080
      DO 20 J=1,3                                                        KK 00090
   20 D(I,J)=0.0D0                                                       KK 00100
      IF(NN.EQ.0) GO TO 10                                               KK 00110
C     FORM ELASTIC MATRIX FOR PLANE STRAIN                               KK 00120
      ER=E/((ON+PR)*(ON-TW*PR))                                          KK 00130
      D(1,1)=ER*(ON-PR)                                                  KK 00140
      D(1,2)=ER*PR                                                       KK 00150
      D(2,1)=D(1,2)                                                      KK 00160
      D(2,2)=D(1,1)                                                      KK 00170
      D(3,3)=ER*(.5D0-PR)                                                KK 00180
      RETURN                                                             KK 00190
C     FORM ELASTIC MATRIX FOR PLANE STRESS                               KK 00200
   10 ER=E/(ON-PR**2)                                                    KK 00210
      D(1,1)=ER                                                          KK 00220
      D(1,2)=ER*PR                                                       KK 00230
      D(2,1)=D(1,2)                                                      KK 00240
      D(2,2)=D(1,1)                                                      KK 00250
      D(3,3)=ER*(1.-PR)/TW                                               KK 00260
      RETURN                                                             KK 00270
      END                                                                KK 00280
```

```
      SUBROUTINE QUAD5(STK,AK,B,NS,N,NGP)                                LL 00010
C     QUAD5 IS A NUMERICAL INTEGRATION SUBROUTINE THAT FORMS THE STIFFNESSLL 00020
C     MATRIX FOR ANY OF THE THREE QUADRILATERAL ELEMENTS IN THIS FAMILY.  LL 00030
C     GAUSSIAN QUADRATURE WITH A MAXIMUM OF FIVE ORDINATES IS USED.       LL 00040
C     CODED BY PROFESSOR GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL   LL 00050
C                                                                         LL 00060
C     THE SUBROUTINE IS CALLED WITH THE FOLLOWING STATEMENT:              LL 00070
C     CALL QUAD5(STK,AK,SS,SN,NS,N,NGP)                                   LL 00080
C     THE CALLING PROGRAM MUST CONTAIN THE FOLLOWING COMMON STATEMENT:    LL 00090
C     COMMON COORD(12,2),ELAST(3,3)                                       LL 00100
C          STK  IS AN ARRAY DIMENSIONED NXN THAT WILL CONTAIN THE STIFFNESLL 00110
C               MATRIX UPON RETURN TO THE CALLING PROGRAM                 LL 00120
C          SS AND SN ARE THE STRESS AND STRAIN VERSUS NODAL POINT DISPLACELL 00130
C               TRANSFORMATION MATRIX, BOTH ARE DIMENSIONED NSXN WHERE    LL 00140
C               NS IS 3*NPT AND N IS 2*NPT                                LL 00150
C          AK   IS AN ARRAY DIMENSIONED NXN REQUIRED IN FORMK             LL 00160
C          B    IS AN ARRAY DIMENSIONED 2XN REQUIRED IN FORMK             LL 00170
C                                                                         LL 00180
C          NGP IS THE NUMBER OF GAUSS POINTS UED IN THE INTEGRATION       LL 00190
C                                                                         LL 00200
C          COORD(12,2) IS AN ARRAY CONTAINING THE COORDINATES OF ALL      LL 00210
C                      THE NODAL POINTS OF THE ELEMENT TO BE CONSTRUCTED  LL 00220
C          ELAST(3,3)  IS AN ARRAY CONTAINING ALL THE ELASTIC CONSTANTS   LL 00230
C                      OF THE ELEMENT, THE ORDER OF THE STRESS COMPONENTS LL 00240
C                      IS: SIGMAX,SIGMAY,TAUXY                            LL 00250
C                                                                         LL 00260
C                                                                         LL 00270
C                ETA                    ETA                 ETA         LL 00280
C                 +                      +                   +          LL 00290
C           U2----+-----U1         U3----U2---U1       U4--U3+-U2--U1    LL 00300
C           |     +     |          |     +    |        |     +     |     LL 00310
C           |     +     |          |     +    |        |     +     |     LL 00320
C           |     +     |          |     +    |        |     +     |     LL 00330
C           |     +     |          |     +    |        U5    +     U12   LL 00340
C           |     +     |          |     +    |        |     +     |     LL 00350
C           |     ++++++++XI       U4    ++++++U8++XI  |     ++++++++XI  LL 00360
C           |           |          |          |        |           |     LL 00370
C           |           |          |          |        U6          U11   LL 00380
C           |           |          |          |        |           |     LL 00390
C           |           |          |          |        |           |     LL 00400
C           |           |          |          |        |           |     LL 00410
C           U3----------U4         U5----U6----U7       U7--U8--U9--U10   LL 00420
C                                                                         LL 00430
      IMPLICIT REAL*8 (A-H,O-Z)                                           LL 00440
      DIMENSION X2(2),X3(3),X4(4),X5(5),A2(2),A3(3),A4(4),A5(5)           LL 00450
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      DIMENSION XYL(4,2),XYQ(8,2),XYC(12,2)                                  LL 00460
      DIMENSION AK(16,16),STK(16,16),B(3,16)                                 LLW00470
      DIMENSION XI(5),AI(5),AIA(5,5)                                         LLW00480
      COMMON COORD(12,2),ELAST(3,3),SS(24,16),SN(24,16)                      LLW00490
      DATA X2/.57735026918962600,-.57735026918962600/                        LL 00500
      DATA A2/1.0D0,1.0D0/                                                   LL 00510
      DATA X3/.77459666924148300,0.0D0,-.77459666924148300/                  LL 00520
      DATA A3/.55555555555555600,.88888888888888900,.55555555555555600/ LL 00530
      DATA X4/.86113631159400D0,.33998104358480D0,-.33998104358480D0,-.8611LL 00540
     136311594000/                                                           LL 00550
      DATA A4/.34785484513740D0,.65214515486250D0,.65214515486250D0,.347854LL 00560
     18451374D0/                                                             LL 00570
      DATA X5/.90617934593860D0,.53846931010560D0,0.0D0,-.53846931010560D0,LL 00580
     1-.90617934593860D0/                                                    LL 00590
      DATA A5/.23692688505610D0,.47862867049930D0,.56888888888830D0,.478628LL 00600
     16704993D0,.23692688505610D0/                                           LL 00610
      DATA XYL/1.0D0,-1.0D0,-1.0D0,1.0D0,1.0D0,1.0D0,-1.0D0,-1.0D0/      LL 00620
      DATA XYQ/1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0,1.0D0,1.0D0,1.0D0,LL 00630
     11.0D0,1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0/                          LL 00640
      DATA XYC/1.0D0,.33333333333330D0,-.33333333333330D0,-1.0D0,-1.0D0,-1LL 00650
     1.0D0,-1.0D0,-.33333333333330D0,.33333333333330D0,1.0D0,1.0D0,1.0D0,1LL 00660
     2.0D0,1.0D0,1.0D0,1.0D0,.33333333333330D0,-.33333333333330D0,-1.0D0,-LL 00670
     31.0D0,-1.0D0,-1.0D0,-.33333333333330D0,.33333333333330D0/              LL 00680
      IF(NGP.EQ.0) NGP=3                                                     LL 00690
      DO 100 I=1,N                                                           LL 00700
      DO 100 J=1,N                                                           LL 00710
  100 STK(I,J)=0.0D0                                                         LL 00720
      JUMP=NGP-1                                                             LL 00730
      DO 140 I=1,NGP                                                         LL 00740
      GO TO (132,133,134,135),JUMP                                           LL 00750
  132 XI(I)=X2(I)                                                            LL 00760
      AI(I)=A2(I)                                                            LL 00770
      GO TO 140                                                              LL 00780
  133 XI(I)=X3(I)                                                            LL 00790
      AI(I)=A3(I)                                                            LL 00800
      GO TO 140                                                              LL 00810
  134 XI(I)=X4(I)                                                            LL 00820
      AI(I)=A4(I)                                                            LL 00830
      GO TO 140                                                              LL 00840
  135 XI(I)=X5(I)                                                            LL 00850
      AI(I)=A5(I)                                                            LL 00860
  140 CONTINUE                                                               LL 00870
      DO 150 I=1,NGP                                                         LL 00880
      DO 150 J=1,NGP                                                         LL 00890
  150 AIA(I,J)=AI(I)*AI(J)                                                   LL 00900
      DO 200 I=1,NGP                                                         LL 00910
      X=XI(I)                                                                LL 00920
      DO 200 J=1,NGP                                                         LL 00930
```

```
      Y=XI(J)                                                          LL 00940
      CALL FORMK(AK,X,Y,B,N)                                           LL 00950
      AIAIJ=AIA(I,J)                                                   LL 00960
      DO 200 K=1,N                                                     LL 00970
      DO 200 L=1,N                                                     LL 00980
  200 STK(K,L)=STK(K,L)+AIAIJ*AK(K,L)                                  LL 00990
      DO 300 I=1,N                                                     LL 01000
      DO 300 J=I,N                                                     LL 01010
      STK(I,J)=(STK(I,J)+STK(J,I))*0.5D0                               LL 01020
  300 STK(J,I)=STK(I,J)                                                LL 01030
      NPT=N/2                                                          LL 01040
      IGO=NPT/4                                                        LL 01050
      DO 1000 I=1,NPT                                                  LL 01060
      GO TO (400,500,600),IGO                                          LL 01070
  400 X=XYL(I,1)                                                       LL 01080
      Y=XYL(I,2)                                                       LL 01090
      GO TO 700                                                        LL 01100
  500 X=XYQ(I,1)                                                       LL 01110
      Y=XYQ(I,2)                                                       LL 01120
      GO TO 700                                                        LL 01130
  600 X=XYC(I,1)                                                       LL 01140
      Y=XYC(I,2)                                                       LL 01150
  700 CALL FORMK(AK,X,Y,B,N)                                           LL 01160
      DO 800 L=1,3                                                     LL 01170
      DO 800 M=1,N                                                     LL 01180
      AK(L,M)=0.0D0                                                    LL 01190
      DO 800 NN=1,3                                                    LL 01200
  800 AK(L,M)=AK(L,M)+ELAST(L,NN)*B(NN,M)                              LL 01210
      II=3*(I-1)                                                       LL 01220
      DO 900 J=1,3                                                     LL 01230
      IJ=II+J                                                          LL 01240
      DO 900 K=1,N                                                     LL 01250
      SS(IJ,K)=AK(J,K)                                                 LL 01260
  900 SN(IJ,K)=B(J,K)                                                  LL 01270
 1000 CONTINUE                                                         LL 01280
      RETURN                                                           LL 01290
      END                                                              LL 01300
```

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      SUBROUTINE FORMK(AK,X,Y,B,N)                                     MM 00010
C THIS SUBROUTINE FORMS THE STIFFNESS AND THE B MATRIX AS FUNCTIONS OF  MM 00020
C XI AND ETA FOR THE THREE DIFFERENT QUADRILATERAL ELEM. IN THIS FAMILY MM 00030
C                                                                      MM 00040
      IMPLICIT REAL*8 (A-H,O-Z)                                        MM 00050
      DIMENSION AJ(2,2),AJIN(2,2),DNX(2,8),W1(2,12),B(3,16)            MMW00060
```

```
     1,B1(16,3),AK(16,16)                                                    MMW00070
      COMMON COORD(12,2),ELAST(3,3),SS(24,16),SN(24,16)                      MMW00080
C INITIALISE                                                                 MM 00090
      ZERO=0.0D0                                                             MM 00100
      DO 10 I=1,3                                                            MM 00110
      DO 10 J=1,N                                                            MM 00120
   10 B(I,J)=ZERO                                                            MM 00130
      ONE=1.0D0                                                              MM 00140
      TWO=2.0D0                                                              MM 00150
      FOUR=4.0D0                                                             MM 00160
      NPT=N/2                                                                MM 00170
C                                                                            MM 00180
C FORM (2XNPT) MATRIX OF DERIV. OF THE INTERP. FUNCT. WRT XI AND ETA         MM 00190
C                                                                            MM 00200
      IGO=N/8                                                                MM 00210
      GO TO (20,30,40),IGO                                                   MM 00220
C                                                                            MM 00230
CC    LINEAR FUNCTIONS                                                       MM 00240
C                                                                            MM 00250
   20 W1(1,3)=-(ONE-Y)/FOUR                                                  MM 00260
      W1(1,4)=-W1(1,3)                                                       MM 00270
      W1(1,1)= (ONE+Y)/FOUR                                                  MM 00280
      W1(1,2)=-W1(1,1)                                                       MM 00290
      W1(2,3)=-(ONE-X)/FOUR                                                  MM 00300
      W1(2,4)=-(ONE+X)/FOUR                                                  MM 00310
      W1(2,1)=-W1(2,4)                                                       MM 00320
      W1(2,2)=-W1(2,3)                                                       MM 00330
      GO TO 50                                                               MM 00340
   30 CONTINUE                                                               MM 00350
C                                                                            MM 00360
CC    QUADRATIC FUNCTIONS                                                    MM 00370
C                                                                            MM 00380
      TXPY=TWO*X+Y                                                           MM 00390
      TXMY=TWO*X-Y                                                           MM 00400
      TYPX=TWO*Y+X                                                           MM 00410
      TYMX=TWO*Y-X                                                           MM 00420
      OMY=ONE-Y                                                              MM 00430
      OPY=ONE+Y                                                              MM 00440
      OPX=ONE+X                                                              MM 00450
      OMX=ONE-X                                                              MM 00460
      W1(1,5)=OMY*TXPY/FOUR                                                  MM 00470
      W1(1,6)=-OMY*X                                                         MM 00480
      W1(1,7)=OMY*TXMY/FOUR                                                  MM 00490
      W1(1,8)=OPY*OMY/TWO                                                    MM 00500
      W1(1,1)=OPY*TXPY/FOUR                                                  MM 00510
      W1(1,2)=-OPY*X                                                         MM 00520
      W1(1,3)= OPY*TXMY/FOUR                                                 MM 00530
      W1(1,4)=-OPY*OMY/TWO                                                   MM 00540
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      W1(2,5)=OMX*TYPX/FOUR                                                  MM 00550
      W1(2,6)=-OPX*OMX/TWO                                                   MM 00560
      W1(2,7)=OPX*TYMX/FOUR                                                  MM 00570
      W1(2,8)=-Y*OPX                                                         MM 00580
      W1(2,1)=OPX*TYPX/FOUR                                                  MM 00590
      W1(2,2)=OPX*OMX/TWO                                                    MM 00600
      W1(2,3)=OMX*TYMX/FOUR                                                  MM 00610
      W1(2,4)=-Y*OMX                                                         MM 00620
      GO TO 50                                                               MM 00630
   40 CONTINUE                                                               MM 00640
C                                                                            MM 00650
CCC   CUBIC FUNCTIONS                                                        MM 00660
C                                                                            MM 00670
      OPX=ONE+X                                                              MM 00680
      OPY=ONE+Y                                                              MM 00690
      OMX=ONE-X                                                              MM 00700
      OMY=ONE-Y                                                              MM 00710
      TPTMNX=3.0D0+2.0D0*X-9.0D0*X*X                                         MM 00720
      TMTMNX=3.0D0-2.0D0*X-9.0D0*X*X                                         MM 00730
      TPTMNY=3.0D0+2.0D0*Y-9.0D0*Y*Y                                         MM 00740
      TMTMNY=3.0D0-2.0D0*Y-9.0D0*Y*Y                                         MM 00750
      TYPO=3.0D0*Y+ONE                                                       MM 00760
      TYMO=3.0D0*Y-ONE                                                       MM 00770
      TXPO=3.0D0*X+ONE                                                       MM 00780
      TXMO=3.0D0*X-ONE                                                       MM 00790
      TMTPEX=10.0D0-27.0D0*X*X+18.0D0*X-9.0D0*Y*Y                            MM 00800
      TMTMEX=10.0D0-27.0D0*X*X-18.0D0*X-9.0D0*Y*Y                            MM 00810
      TMNPEY= 10.0D0-9.0D0*X*X+18.0D0*Y-27.0D0*Y*Y                           MM 00820
      TMNMEY= 10.0D0-9.0D0*X*X-18.0D0*Y-27.0D0*Y*Y                           MM 00830
      TTT=32.0D0                                                             MM 00840
      TTN=32.0D0/9.0D0                                                       MM 00850
      W1(1,7)= OMY*TMTPEX/TTT                                                MM 00860
      W1(1,8)=-TPTMNX*OMY/TTN                                                MM 00870
      W1(1,9)=TMTMNX*OMY/TTN                                                 MM 00880
      W1(1,10)=-OMY*TMTMEX/TTT                                               MM 00890
      W1(1,11)=-OPY*OMY*TYMO/TTN                                             MM 00900
      W1(1,12)=OPY*OMY*TYPO/TTN                                              MM 00910
      W1(1,1)=-OPY*TMTMEX/TTT                                                MM 00920
      W1(1,2)=TMTMNX*OPY/TTN                                                 MM 00930
      W1(1,3)=-TPTMNX*OPY/TTN                                                MM 00940
      W1(1,4) =OPY*TMTPEX/TTT                                                MM 00950
      W1(1,5) =-OPY*OMY*TYPO/TTN                                             MM 00960
      W1(1,6) = OPY*OMY*TYMO/TTN                                             MM 00970
      W1(2,7) = OMX*TMNPEY/TTT                                               MM 00980
      W1(2,8)= OPX*OMX*TXMO/TTN                                              MM 00990
      W1(2,9)=-OPX*OMX*TXPO/TTN                                              MM 01000
      W1(2,10) = OPX*TMNPEY/TTT                                              MM 01010
      W1(2,11)=-TPTMNY*OPX/TTN                                               MM 01020
```

```
      W1(2,12)= TMTMNY*OPX/TTN                                          MM 01030
      W1(2,1) =-OPX*TMNMEY/TTT                                          MM 01040
      W1(2,2)=OPX*OMX*TXPO/TTN                                          MM 01050
      W1(2,3)=-OPX*OMX*TXMO/TTN                                         MM 01060
      W1(2,4) =-OMX*TMNMEY/TTT                                          MM 01070
      W1(2,5) =OMX*TMTMNY/TTN                                           MM 01080
      W1(2,6) =-OMX*TPTMNY/TTN                                          MM 01090
   50 CONTINUE                                                          MM 01100
C                                                                       MM 01110
CC FORM JACOBIAN OF THE TRANSFORMATION IN AJ(I,J)                       MM 01120
C                                                                       MM 01130
      DO 100 I=1,2                                                      MM 01140
      DO 100 J=1,2                                                      MM 01150
      AJ(I,J)=ZERO                                                      MM 01160
      DO 100 K=1,NPT                                                    MM 01170
  100 AJ(I,J)=AJ(I,J)+W1(I,K)*COORD(K,J)                                MM 01180
C                                                                       MM 01190
CC CALCULATE DETERMINANT OF THE JACOBIAN                                MM 01200
C                                                                       MM 01210
      DTJ=AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)                               MM 01220
C                                                                       MM 01230
CC INVERT JACOBIAN IN AJIN                                              MM 01240
C                                                                       MM 01250
      AJIN(1,1)=AJ(2,2)/DTJ                                             MM 01260
      AJIN(1,2)=-AJ(1,2)/DTJ                                            MM 01270
      AJIN(2,1)=-AJ(2,1)/DTJ                                            MM 01280
      AJIN(2,2)=AJ(1,1)/DTJ                                             MM 01290
C                                                                       MM 01300
CC FORM (2XNPT) MATRIX OF DERIVATIVES OF INTERP. FUNCT WRT X AND Y      MM 01310
C                                                                       MM 01320
      DO 200 I=1,2                                                      MM 01330
      DO 200 J=1,NPT                                                    MM 01340
      DNX(I,J)=ZERO                                                     MM 01350
      DO 200 K=1,2                                                      MM 01360
  200 DNX(I,J)=DNX(I,J)+AJIN(I,K)*W1(K,J)                               MM 01370
C                                                                       MM 01380
CC FORM B(I,J) MATRIX                                                   MM 01390
C                                                                       MM 01400
      DO 300 I=1,NPT                                                    MM 01410
      IOD=2*I-1                                                         MM 01420
      IEV=2*I                                                           MM 01430
      B(1,IOD)=DNX(1,I)                                                 MM 01440
      B(2,IEV)=DNX(2,I)                                                 MM 01450
      B(3,IEV)=DNX(1,I)                                                 MM 01460
  300 B(3,IOD)=DNX(2,I)                                                 MM 01470
C                                                                       MM 01480
CC FORM STIFFNESS BY THE CONGRUENT TRANSFORMATION  BT*D*B               MM 01490
C                                                                       MM 01500
```

```
      DO 500 I=1,N                                                      MM 01510
      DO 500 J=1,3                                                      MM 01520
      B1(I,J)=ZERO                                                      MM 01530
      DO 500 K=1,3                                                      MM 01540
  500 B1(I,J)=B1(I,J)+B (K,I)*ELAST(K,J)                                MM 01550
      DO 600 I=1,N                                                      MM 01560
      DO 600 J=1,N                                                      MM 01570
      AK(I,J)=ZERO                                                      MM 01580
      DO 600 K=1,3                                                      MM 01590
  600 AK(I,J)=AK(I,J)+B1(I,K)*B(K,J)                                    MM 01600
      DO 700 I=1,N                                                      MM 01610
      DO 700 J=1,N                                                      MM 01620
      AK(I,J)=((AK(I,J)+AK(J,I))/2.0D0)*DTJ                             MM 01630
  700 AK(J,I)=AK(I,J)                                                   MM 01640
      RETURN                                                            MM 01650
      END                                                               MM 01660
```

```
      SUBROUTINE LDLT(N,M)                                              NN 00010
C     THIS SUBROUTINE DECOMPOSES THE COEFFICIENT MATRIX A* OF THE LINEAR NN 00020
C     BANDED SYMMETRICAL SYSTEM (A*)(X) = (B), INTO THE PRODUCT (L)(D)(LT)NN 00030
C     THE UPPER BAND ONLY IS USED, AND IT MUST BE STORED IN A RECTANGULAR NN 00040
C     ARRAY (A) WITH ALL ITS DIAGONAL ELEMENTS IN THE FIRST COLUMN      NN 00050
C     MATRIX (A) IS DESTROYED IN THE PROCESS AND THE RESULTS RETURNED IN NN 00060
C     THE SAME ARRAY. MATRIX (D) IS PUT IN THE FIRST COLUMN AND THE REST NN 00070
C     OF THE MATRIX CONTAINS THE ELEMENTS OF THE UNIT TRIANGULAR MATRIX (LNN 00080
C     IN BANDED FORM WITHOUT ITS IMPLIED UNIT DIAGONAL ELEMENTS.        NN 00090
C                                                                       NN 00100
C         A  IS THE NAME OF THE COEFFICIENT MATRIX                      NN 00110
C         N  IS THE NUMBER OF EQUATIONS IN THE SYSTEM                   NN 00120
C         M  IS THE HALF BAND WIDTH OF THE SYSTEM                       NN 00130
C     CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972       NN 00140
C                                                                       NN 00150
      IMPLICIT REAL*8 (A-H,O-Z)                                         NN 00160
      COMMON/SOL/A( 908,64),B( 908),ABGN                                NNW00170
      AN=N                                                              NN 00180
      APZRO=ABGN*1.0D-30/AN                                             NN 00190
      AVSN=APZRO/ABGN                                                   NN 00200
      ZRO=0.0D0                                                         NN 00210
      NM=N-1                                                            NN 00220
      DO 300 I=1,NM                                                     NN 00230
      DIAG=A(I,1)                                                       NN 00240
      IF(DIAG.EQ.ZRO) GO TO 400                                         NN 00250
      IF(DIAG.LE.APZRO) WRITE(6,2000) I                                 NN 00260
      DO 100 J=2,M                                                      NN 00270
```

```
      IF( . DABS(A(I,J)).LE.AVSN) A(I,J)=0.0D0                                  NN 00280
  100 A(I,J)=A(I,J)/DIAG                                                        NN 00290
      DO 300 J=2,M                                                              NN 00300
      L=I+J-1                                                                   NN 00310
      IF(L.GT.N) GO TO 300                                                      NN 00320
      AA=A(I,J)*DIAG                                                            NN 00330
      IF(AA.EQ.ZRO) GO TO 300                                                   NN 00340
      DO 200 K=J,M                                                              NN 00350
      ML=1+K-J                                                                  NN 00360
  200 A(L,ML)=A(L,ML)-AA*A(I,K)                                                 NN 00370
  300 CONTINUE                                                                  NN 00380
      RETURN                                                                    NN 00390
  400 WRITE(6,1000) I                                                           NN 00400
      STOP                                                                      NN 00410
 1000 FORMAT(//,5X,' MATRIX IS SINGULAR ',I15)                                  NN 00420
 2000 FORMAT(//,5X,' WARNING  MATRIX IS NEARLY SINGULAR  EXECUTION CONTINN 00430
     1NUES ', I15,/,5X,' RESIDUAL ITERATION MAY YIELD ERRONEOUS RESULTS NN 00440
     2  '//)                                                                   NN 00450
      END                                                                       NN 00460
      SUBROUTINE SOLV(N,M)                                                      00 00010
C    THIS SUBROUTINE PERFORMS A FORWARD SUBSTITUTION, FOLLOWED BY A           00 00020
C    BACKWARD SUBSTITUTION FOR THE SYSTEM (A*) X= (B), AFTER THE              00 00030
C    MATRIX (A) HAS BEEN DECOMPOSED BY THE SUBROUTINE LDLDT                   00 00040
C                                                                             00 00050
C         A  IS THE NAME OF THE DECOMPOSED MATRIX OF COEFFICIENT              00 00060
C         B  IS THE NAME OF THE COLUMN LOAD VECTOR OR RIGHT HAND MEMBER       00 00070
C            THE ANSWERS ARE RETURNED IN THIS VECTOR UPON EXIT                00 00080
C         N  IS THE NUMBER OF EQUATIONS IN THE SYSTEM                         00 00090
C         M  IS THE HALF BAND WIDTH OF THE SYTEM                              00 00100
C    AFTER ONE CALL TO LDLT , ONE CALL OF SLV PER LOAD VECTOR IS REQUIREDOO 00110
C    CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972              00 00120
C                                                                             00 00130
      IMPLICIT REAL*8 (A-H,O-Z)                                               00 00140
      COMMON/SOL/A( 908,64),B( 908),ABGN                                      00W00150
      NM=N-1                                                                  00 00160
      DO 100 I=1,NM                                                           00 00170
      BI=B(I)                                                                 00 00180
      DO 100 J=2,M                                                            00 00190
      L=I+J-1                                                                 00 00200
      IF(L.GT.N) GO TO 100                                                    00 00210
      B(L)=B(L)-A(I,J)*BI                                                     00 00220
  100 CONTINUE                                                                00 00230
      DO 200 I=1,N                                                            00 00240
  200 B(I)=B(I)/A(I,1)                                                        00 00250
      DO 300 L=2,N                                                            00 00260
      IR=N-L+1                                                                00 00270
      BIRP=B(IR+1)                                                            00 00280
      DO 300 J=2,M                                                            00 00290
```

```
      IRO=IR-J+2                                                        OO 00300
      IF(IRO.LE.0) GO TO 300                                            OO 00310
      B(IRO)=B(IRO)-A(IRO,J)*BIRP                                       OO 00320
  300 CONTINUE                                                          OO 00330
      RETURN                                                            OO 00340
      END                                                               OO 00350



      SUBROUTINE STRESS(NPEL)                                           PP 00010
      IMPLICIT REAL*8 (A-H,O-Z)                                         PP 00020
      COMMON/TITL/TITLE(12)                                             PP 00030
      COMMON/SOL/BGK( 908,64), DISP( 908),ABGN                          PPW00040
      COMMON/STAT/WR,DR,TH,W,X(33),Y(33),EXPX(48),EXPY(48)              PPW00050
      COMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(135,14),NSTRES,NGP,     PPW00060
     1LM(8),LJT(8),NTLD,LTBY,NTIN                                       PPW00070
      COMMON/CORD1/COARD(454,2),CLOAD(454,2),ELCON(1,4),NBC(454,2)      PPW00080
      COMMON COORD(12,2),ELAST(3,3),SS(24,16),SN(24,16)                 PPW00090
      DIMENSION SSJNT(454,7),SNJNT(454,7),SSEL(24),SNEL(24),            PPW00100
     1DSPEL(24)                                                         PPW00110
      EQUIVALENCE (BGK(1,1),SSJNT(1,1)),(BGK(1,5),SNJNT(1,1)),(BGK(1,9),PP 00120
     1SSEL(1)),(BGK(1,10),SNEL(1)),(BGK(1,11),DSPEL(1))                 PP 00130
      PI=3.141592653590D0                                               PP 00140
      ZRO=0.000                                                         PP 00150
      REWIND 12                                                         PP 00160
      DO 50 I=1,NJT                                                     PP 00170
      DO 50 J=1,7                                                       PP 00180
      SNJNT(I,J)=ZRO                                                    PP 00190
   50 SSJNT(I,J)=ZRO                                                    PP 00200
      DO 300 I=1,NEL                                                    PP 00210
      READ(12) SS,SN                                                    PP 00220
      DO 100 J=1,NPEL                                                   PP 00230
      JA=NCON(I,J)*2-1                                                  PP 00240
      JJ=J*2-1                                                          PP 00250
      DSPEL(JJ)=DISP(JA)                                                PP 00260
  100 DSPEL(JJ+1)=DISP(JA+1)                                            PP 00270
      NJLM=NPEL*2                                                       PP 00280
      NPELT=NPEL*3                                                      PP 00290
      DO 200 I1=1,NPELT                                                 PP 00300
      SSEL(I1)=ZRO                                                      PP 00310
      SNEL(I1)=ZRO                                                      PP 00320
      DO 200 J1=1,NJLM                                                  PP 00330
      SSEL(I1)=SSEL(I1)+SS(I1,J1)*DSPEL(J1)                             PP 00340
  200 SNEL(I1)=SNEL(I1)+SN(I1,J1)*DSPEL(J1)                             PP 00350
      IF(NMAT.EQ.1) GO TO 220                                           PP 00360
      WRITE(6,4000) I                                                   PP 00370
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```
      ILO=0                                                                  PP 00380
      IUP=0                                                                  PP 00390
      DO 210 J=1,NPEL                                                        PP 00400
      IJT=NCON(I,J)                                                          PP 00410
      ILO=IUP+1                                                              PP 00420
      IUP=ILO+2                                                              PP 00430
  210 WRITE(6,5000) IJT,(SSEL(K),K=ILO,IUP),(SNEL(L),L=ILO,IUP)              PP 00440
  220 CONTINUE                                                               PP 00450
 5000 FORMAT(5X,I5,6G12.4)                                                   PP 00460
 4000 FORMAT(//,2X,' S-T-R-E-S-S-E-S  /  S-T-R-A-I-N-S  FOR  ELEMENT',I3PP 00470
     1,      //,5X,' JOINT',1X,'SIG X ',6X,'SIG Y ',7X,'TAU ',7X,'EPS XPP 00480
     2',7X,'EPS Y', 7X,'GAMMA ',//)                                          PP 00490
      ICT=0                                                                  PP 00500
      DO 300 I2=1,NPEL                                                       PP 00510
      I2A=NCON(I,I2)                                                         PP 00520
      DO 250 J2=1,3                                                          PP 00530
      ICT = ICT + 1                                                          PP 00540
      SSJNT(I2A,J2)=SSEL(ICT)+SSJNT(I2A,J2)                                  PP 00550
  250 SNJNT(I2A,J2)=SNEL(ICT)+SNJNT(I2A,J2)                                  PP 00560
  300 SNJNT(I2A,7)=SNJNT(I2A,7)+1.                                           PP 00570
      DO 500 I3=1,NJT                                                        PP 00580
      SCNT=SNJNT(I3,7)                                                       PP 00590
      DO 400 J3=1,3                                                          PP 00600
      SSJNT(I3,J3)=SSJNT(I3,J3)/SCNT                                         PP 00610
  400 SNJNT(I3,J3)=SNJNT(I3,J3)/SCNT                                         PP 00620
      GAU=SSJNT(I3,3)                                                        PP 00630
      SXPSY=(SSJNT(I3,1)+SSJNT(I3,2))/2.0D0                                  PP 00640
      SXMSY=(SSJNT(I3,1)-SSJNT(I3,2))/2.0D0                                  PP 00650
      AINT=DSQRT(SXMSY*SXMSY+GAU*GAU)                                        PP 00660
      SSJNT(I3,4)=SXPSY+AINT                                                 PP 00670
      SSJNT(I3,5)=SXPSY-AINT                                                 PP 00680
      SSJNT(I3,6)=(SSJNT(I3,4)-SSJNT(I3,5))/2.0D0                            PP 00690
      IF(   DABS(SXMSY).LE.0.00001D0) GO TO 420                              PP 00700
      RAN=-GAU/SXMSY                                                         PP 00710
      IF(   DABS(RAN).LE.0.01D0) GO TO 450                                   PP 00720
      RAN=   DATAN(RAN)                                                      PP 00730
      GO TO 450                                                              PP 00740
  420 RAN=DSIGN(PI,GAU)/2.0D0                                                PP 00750
  450 SSJNT(I3,7)=RAN*180.0D0/(2.0D0*PI)                                     PP 00760
      GAU=SNJNT(I3,3)                                                        PP 00770
      SXPSY=(SNJNT(I3,1)+SNJNT(I3,2))/2.0D0                                  PP 00780
      SXMSY=(SNJNT(I3,1)-SNJNT(I3,2))/2.0D0                                  PP 00790
      AINT=DSQRT(SXMSY*SXMSY+GAU*GAU)                                        PP 00800
      SNJNT(I3,4)=SXPSY+AINT                                                 PP 00810
      SNJNT(I3,5)=SXPSY-AINT                                                 PP 00820
      SNJNT(I3,6)=(SNJNT(I3,4)-SNJNT(I3,5))/2.0D0                            PP 00830
  500 SNJNT(I3,7)=SSJNT(I3,7)                                                PP 00840
      WRITE(6,1000)                                                          PP 00850
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      DO 600 I=1,NJT                                                    PP 00860
      WRITE(6,2000) I,(SSJNT(I,J),J=1,7)                                PP 00870
  600 CONTINUE                                                          PP 00880
      DO 800 I=1,25                                                     PP 00890
      Y(I)=SSJNT(I,1)                                                   PP 00900
  800 CONTINUE                                                          PP 00910
 6218 FORMAT(2D25.16)                                                   PP 00920
      Y(14)=SSJNT(1,1)                                                  PP 00930
 2000 FORMAT(5X,I5,6G12.4,5X,1G12.4)                                    PP 00940
      WRITE(6,3000)                                                     PP 00950
      DO 700 I=1,NJT                                                    PP 00960
      WRITE(6,2000) I,(SNJNT(I,J),J=1,6),SSJNT(I,7)                     PP 00970
  700 CONTINUE                                                          PP 00980
 1000 FORMAT(///,1X,' A-V-E-R-A-G-E  S-T-R-E-S-S-E-S  AT  THE  JOINTS',  PP 00990
     1       //,5X,' JOINT',1X,'SIG X ',6X,'SIG Y ',7X,'TAU ',7X,'S.MAXPP 01000
     2',7X,'S.MIN', 7X,'T.MAX ',12X,'  TET(DEG.)'//)                    PP 01010
 3000 FORMAT(///,1X,' A-V-E-R-A-G-E  S-T-R-A-I-N-S  AT  THE  JOINTS',   PP 01020
     1       //,5X,' JOINT',1X,'EPS X ',6X,'EPS Y ',7X,'GAMMA',7X,'E.MAXPP 01030
     2',7X,'E.MIN', 7X,'G.MAX ',12X,'  TET(DEG.)',//)                   PP 01040
      RETURN                                                            PP 01050
      END                                                               PP 01060
```

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      SUBROUTINE BCOND(NBAND)                                           QQ 00010
      IMPLICIT REAL*8 (A-H,O-Z)                                         QQ 00020
      COMMON/SOL/BGK( 908,64),ALOAD( 908),ABGN                          QQW00030
      COMMON/UNT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(135,14),NSTRES,NGP,     QQW00040
     1LM(6),LJT(8),NTLD,LTBY,NTIN                                       QQW00050
      COMMON/CORD1/COARD(454,2),CLOAD(454,2),ELCON(1,4),NBC(454,2)      QQW00060
      COMMON/TITL/TITLE(12)                                             QQ 00070
      PI=3.141592653590                                                 QQ 00080
      NEQ=NJT*2                                                         QQ 00090
      DO 200 I=1,NPBC                                                   QQ 00100
      IJT=NBC(I,1)                                                      QQ 00110
      IEQ=IJT*2-1                                                       QQ 00120
      KODE=NBC(I,2)                                                     QQ 00130
      IDIA=0                                                            QQ 00140
      GO TO (110,120,130,140,150,160,170),KODE                          QQ 00150
  110 IEQ=IEQ+1                                                         QQ 00160
  112 BGK(IEQ,1)=ABGN                                                   QQ 00170
      IF(IDIA.EQ.1) GO TO 132                                           QQ 00180
      GO TO 200                                                         QQ 00190
  120 GO TO 112                                                         QQ 00200
  130 IDIA=1                                                            QQ 00210
      GO TO 112                                                         QQ 00220
```

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132 IDIA=0                                                                      QQ 00230
    GO TO 110                                                                  QQ 00240
140 DXY=CLOAD(IJT,2)                                                           QQ 00250
142 IEQ=IEQ+1                                                                  QQ 00260
143 ALOAD(IEQ)=ALOAD(IEQ)-BGK(IEQ,1)*DXY                                       QQ 00270
    BGK(IEQ,1)=ABGN                                                            QQ 00280
    DO 146 J=2,NBAND                                                           QQ 00290
    IROW=IEQ-J+1                                                               QQ 00300
    ICOL=IEQ-1+J                                                               QQ 00310
    IF(IROW.LT.1) GO TO 144                                                    QQ 00320
    ALOAD(IROW)=ALOAD(IROW)-BGK(IROW,J)*DXY                                    QQ 00330
144 IF(ICOL.GT.NEQ) GO TO 146                                                  QQ 00340
    ALOAD(ICOL)=ALOAD(ICOL)-BGK(IEQ,J)*DXY                                     QQ 00350
146 CONTINUE                                                                   QQ 00360
    IF(IDIA.EQ.1) GO TO 162                                                    QQ 00370
    GO TO 200                                                                  QQ 00380
150 DXY=CLOAD(IJT,1)                                                           QQ 00390
    GO TO 143                                                                  QQ 00400
160 IDIA=1                                                                     QQ 00410
    DXY=CLOAD(IJT,1)                                                           QQ 00420
    GO TO 143                                                                  QQ 00430
162 IDIA=0                                                                     QQ 00440
    GO TO 140                                                                  QQ 00450
170 ALPHA=CLOAD(IJT,2)*PI/180.0D0                                              QQ 00460
    CAL=DCOS(ALPHA)                                                            QQ 00470
    SAL=DSIN(ALPHA)                                                            QQ 00480
    IEQP=IEQ+1                                                                 QQ 00490
    DO 180 J=3,NBAND                                                           QQ 00500
    IROW=IEQ-J+2                                                               QQ 00510
    JM=J-1                                                                     QQ 00520
    IF(IROW.LT.1) GO TO 175                                                    QQ 00530
    A1=BGK(IROW,JM)*CAL+BGK(IROW,J)*SAL                                        QQ 00540
    A2=BGK(IROW,J)*CAL-BGK(IROW,JM)*SAL                                        QQ 00550
    BGK(IROW,JM)=A1                                                            QQ 00560
    BGK(IROW,J )=A2                                                            QQ 00570
175 IF((IEQ+J-1).GT.NEQ) GO TO 180                                             QQ 00580
    A3=BGK(IEQ,J)*CAL+BGK(IEQP,JM)*SAL                                         QQ 00590
    A4=BGK(IEQP,JM)*CAL-BGK(IEQ,J)*SAL                                         QQ 00600
    BGK(IEQ ,J )=A3                                                            QQ 00610
    BGK(IEQP,JM)=A4                                                            QQ 00620
180 CONTINUE                                                                   QQ 00630
    A1=BGK(IEQ,1)*CAL*CAL+BGK(IEQ,2)*CAL*SAL*2.0  +BGK(IEQP,1)*SAL*SALQQ 00640
    A2=BGK(IEQ,2)*(CAL*CAL-SAL*SAL)+(BGK(IEQP,1)-BGK(IEQ,1))*SAL*CAL   QQ 00650
    BGK(IEQ,1)=A1                                                              QQ 00660
    BGK(IEQ,2)=A2                                                              QQ 00670
    BGK(IEQP,1)=ABGN                                                           QQ 00680
200 CONTINUE                                                                   QQ 00690
    RETURN                                                                     QQ 00700
```

```
      END                                                                    QQ 00710



      SUBROUTINE DISPL                                                       RR 00010
      IMPLICIT REAL*8 (A-H,O-Z)                                              RR 00020
      COMMON/MDIM/MEL,MJT,MMT,MBD                                            RR 00030
      COMMON/TITL/TITLE(12)                                                  RR 00040
      COMMON/SOL/BGK( 908,64), DISP( 908),ABGN                               RRW00050
      COMMON/UNIT1/NEL,NJT,NMAT,NCLOAD,NPBC,NCON(135,14),NSTRES,NGP,         RRW00060
     1LM(8),LJT(8),NTLD,LTBY,NTIN                                            RRW00070
      COMMON/CORD1/COORD(454,2),CLOAD(454,2),ELCON(1,4),NBC(454,2)           RRW00080
      DIMENSION REACT(455,2)                                                 RRW00090
      EQUIVALENCE (BGK(1,1),REACT(1,1))                                      RR 00100
      PI=3.1415926535900                                                     RR 00110
      DO 200 I=1,NPBC                                                        RR 00120
      IJT=NBC(I,1)                                                           RR 00130
      IEQ=IJT*2-1                                                            RR 00140
      KODE=NBC(I,2)                                                          RR 00150
        IDIA=0                                                               RR 00160
      DPBC=0.0D0                                                             RR 00170
      GO TO (110,120,130,140,150,160,170),KODE                               RR 00180
  110 IEQ=IEQ+1                                                              RR 00190
      JR=2                                                                   RR 00200
  112 REACT(I,JR)=-DISP(IEQ)*ABGN                                            RR 00210
      DISP(IEQ)=DPBC                                                         RR 00220
      IF(IDIA.EQ.1) GO TO 132                                                RR 00230
      IF(IDIA.EQ.2) GO TO 162                                                RR 00240
      GO TO 200                                                              RR 00250
  120 JR=1                                                                   RR 00260
      GO TO 112                                                              RR 00270
  130 IDIA=1                                                                 RR 00280
      JR=1                                                                   RR 00290
      GO TO 112                                                              RR 00300
  132 IDIA=0                                                                 RR 00310
      GO TO 110                                                              RR 00320
  140 DPBC=CLOAD(IJT,2)                                                      RR 00330
      GO TO 110                                                              RR 00340
  150 JR=1                                                                   RR 00350
      DPBC=CLOAD(IJT,1)                                                      RR 00360
      GO TO 112                                                              RR 00370
  160 IDIA=2                                                                 RR 00380
      JR=1                                                                   RR 00390
      DPBC=CLOAD(IJT,1)                                                      RR 00400
      GO TO 112                                                              RR 00410
  162 IDIA=0                                                                 RR 00420
```

```
      CPBC=CLOAD(IJT,2)                                                    RR 00430
      GO TO 110                                                            RR 00440
  170 ALPHA=CLOAD(IJT,2)*PI/180.0D0                                        RR 00450
      CAL=DCOS(ALPHA)                                                      RR 00460
      SAL=DSIN(ALPHA)                                                      RR 00470
      A1=DISP(IEQ)*CAL                                                     RR 00480
      A2=DISP(IEQ)*SAL                                                     RR 00490
      IEQP=IEQ+1                                                           RR 00500
      A3=-DISP(IEQP)*SAL*ABGN                                              RR 00510
      A4= DISP(IEQP)*CAL*ABGN                                              RR 00520
      DISP(IEQ)=A1                                                         RR 00530
      DISP(IEQP)=A2                                                        RR 00540
      REACT(I   ,1)=-A3                                                    RR 00550
      REACT(I   ,2)=-A4                                                    RR 00560
  200 CONTINUE                                                             RR 00570
      MJP=MJT+1                                                            RR 00580
      REACT(MJP,1)=0.0D0                                                   RR 00590
      REACT(MJP,2)=0.0D0                                                   RR 00600
      DO 300 I=1,NPBC                                                      RR 00610
      REACT(MJP,1)=REACT(MJP,1)+REACT(I,1)                                 RR 00620
  300 REACT(MJP,2)=REACT(MJP,2)+REACT(I,2)                                 RR 00630
      RETURN                                                               RR 00640
      END                                                                  RR 00650
```



```
      SUBROUTINE ANS                                                       SS 00010
      IMPLICIT REAL*8 (A-H,O-Z)                                            SS 00020
      COMMON/STAT/WR,DR,TH,W,X(33),Y(33),EXPX(48),EXPY(48)                 SSW00030
      COMMON/INT/NPT,NEL,NPUNCH,NSTOP,NJT,NPLOT,NKIND,NSEL,NGROW,NGCOL,    SS 00031
     1           NCLOAD,NPBC                                               SS 00032
      D=DR*W/WR                                                            SS 00040
      IF(NPUNCH.EQ.2) GO TO 2                                              SS 00050
      SCF=D*Y(14)*1.D-6                                                    SS 00060
      GO TO 3                                                              SS 00070
    2 SCF=D**2*Y(14)/1.3D6                                                 SS 00080
      IF(NPUNCH.EQ.4) SCF = SCF/6.D0                                       SS 00090
    3 SCF=DABS(SCF)                                                        SS 00100
      WRITE(6,1000) SCF                                                    SS 00110
 1000 FORMAT(' ',20X,'SCF=',D15.8,/////////////////)                       SS 00120
      WRITE(7,1001) WR,DR,TH,SCF                                           SS 00130
 1001 FORMAT(5D15.8)                                                       SS 00140
      RETURN                                                               SS 00150
      END                                                                  SS 00160
```

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